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High-energy central exclusive production of the lightest vacuum resonance related to the soft Pomeron



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ARTICLE INFO	ABSTRACT
Article history: Received 2 October 2018 Received in revised form 29 October 2018 Accepted 30 October 2018 Available online 5 November 2018 Editor: W. Haxton	A simple model based on Regge approach is proposed for description of the central exclusive production (CEP) of the light tensor glueball lying on the Regge trajectory of the soft Pomeron. © 2018 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP ³ .

1. Introduction

Reactions of diffractive CEP of light vacuum resonances in high-energy collisions of protons $p + p \rightarrow p + R + p$ (signs "+" denote rapidity gaps) are a valuable source of information on the nonperturbative aspects of strong interaction. At present, they are actively studied by both experimentalists [1,2] and theorists [3,4].

In particular, some of the produced particles may be glueballs, *i.e.*, hadrons with prevailing gluon content, which have not been discovered yet. One of the most promising light tensor glueball candidates is the low-mass resonance of spin 2 related to the Regge trajectory (see Fig. 1) of the soft Pomeron (a Reggeon which dominates in the elastic scattering of protons at ultrahigh energies). In [5], some partial widths of decay to pairs of light mesons were estimated for this resonance (further, we call it $f_2^{(P)}(X)$) in the framework of the Regge-eikonal approach. However, for reliable identification of $f_2^{(P)}(X)$ among other vacuum resonances produced exclusively at the RHIC or the LHC, we need both to know its branching ratios and to estimate the integrated and differential cross-sections of reaction $p + p \rightarrow p + f_2^{(P)}(X) + p$.

The aim of this eprint is to provide such an estimation with the help of the simplest Regge-eikonal model applied earlier to the high-energy elastic scattering [6] and single diffractive dissociation (SDD) [7] of protons.

2. The model

In the region of high values of the collision energy and low values of the proton momentum transfers, the cross-section of exclusive diffractive production of glueball $f_2^{(P)}(X)$ can be represented as

$$\sigma_{p+p \to p+f_{2}^{(P)}(X)+p}^{(\lambda)}(\sqrt{s}) = \frac{1}{2s} \int \frac{|T^{(\lambda)}|^{2} d^{3} p_{1}' d^{3} p_{2}' d^{3} k}{2p_{1}'^{0}(2\pi)^{3} 2p_{2}'^{0}(2\pi)^{3} 2k^{0}(2\pi)^{3}} (2\pi)^{4} \delta^{4}(p_{1}+p_{2}-p_{1}'-p_{2}'-k) \approx \frac{1}{512\pi^{4}s} \int |T^{(\lambda)}(s,\xi_{1},\xi_{2},t_{1},t_{2},\phi)|^{2} \delta(\xi_{1}\xi_{2}s-(\vec{\Delta}_{1\perp}+\vec{\Delta}_{2\perp})^{2}-M_{f_{2}}^{2}) d\xi_{1} d\xi_{2} dt_{1} dt_{2} d\phi,$$
(1)

where p_i and p'_i are the 4-momenta of incoming and outgoing protons, $s = (p_1 + p_2)^2$, vectors $\vec{\Delta}_{i\perp}$ are the transverse components of $\Delta_i \equiv p_i - p'_i$ ($t_i = \Delta_i^2 \approx -|\vec{\Delta}_{i\perp}|^2/(1-\xi_i)$), $k = \Delta_1 + \Delta_2$ is the 4-momentum of the produced tensor state, $\xi_i \ll 1$ are the energy fractions lost by the diffractively scattered protons, λ is the produced particle helicity, ϕ is the angle between $\vec{\Delta}_{1\perp}$ and $\vec{\Delta}_{2\perp}$, M_{f_2} is the produced resonance mass, and $T^{(\lambda)}$ is the full helicity amplitude of the $f_2^{(P)}(X)$ CEP.

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Fig. 1. Expected qualitative behavior of the real part of the Pomeron Regge trajectory in the resonance region.



Fig. 2. The diagrams for the central exclusive production of $f_2^{(P)}(X)$ via two-Pomeron fusion (left) and the single diffractive dissociation of proton at high missing masses (right).

If the produced resonance has a significant decay width Γ_{f_2} , then the following replacement should be made in (1): $\delta(\xi_1\xi_2s - (\vec{\Delta}_{1\perp} + \vec{\Delta}_{2\perp})^2 - M_{f_2}^2) \rightarrow \frac{1}{\pi} \frac{M_{f_2}\Gamma_{f_2}}{(M_{f_2}^2 + (\vec{\Delta}_{1\perp} + \vec{\Delta}_{2\perp})^2 - \xi_1\xi_2s)^2 + M_{f_2}^2\Gamma_{f_2}^2}$. Next, constructing the double Pomeron fusion vertex in terms of independent tensor structures $\Delta_1^{\mu}\Delta_1^{\nu}$, $\Delta_1^{\mu}k^{\nu}$, $k^{\mu}\Delta_1^{\nu}$, $k^{\mu}k^{\nu}$, and $g^{\mu\nu}$,

and taking account of the symmetry, transversality, and tracelessness of the produced glueball helicity states $e_{\mu\nu}^{(\lambda)}(k)$, we come to the expression for the bare helicity amplitudes of the $f_2^{(P)}(X)$ exclusive production:

$$T_{bare}^{(\lambda)}(s, \xi_{1}, \xi_{2}, \vec{\Delta}_{1\perp}, \vec{\Delta}_{2\perp}) = \left(i + \tan \frac{\pi (\alpha_{P}(t_{1}) - 1)}{2}\right) \left(i + \tan \frac{\pi (\alpha_{P}(t_{2}) - 1)}{2}\right) \times \left(\frac{1}{\xi_{1}}\right)^{\alpha_{P}(t_{1})} \left(\frac{1}{\xi_{2}}\right)^{\alpha_{P}(t_{2})} \pi^{2} \alpha_{P}'(t_{1}) \alpha_{P}'(t_{2}) g_{ppP}(t_{1}) g_{ppP}(t_{2}) g_{PPf_{2}}(t_{1}, t_{2}) \frac{\Delta_{1}^{\mu} \Delta_{1}^{\nu}}{s_{0}} e_{\mu\nu}^{(\lambda)}(k),$$
(2)

where $\alpha_P(t)$ is the Regge trajectory of the Pomeron, $g_{ppP}(t)$ is the Pomeron coupling to proton, $s_0 = 1 \text{ GeV}^2$ is the unit of measurement,

and $g_{PPf_2}(t_1, t_2)$ is the structure function related to the structure $\Delta_1^{\mu} \Delta_1^{\nu}$ in the double Pomeron fusion vertex. The factors $\pi \alpha'_p$ are singled out within the Regge residue for the same reasons as in the cases of elastic scattering [6] and high-missing-mass SDD [7]. Comparing the diagram for the $f_2^{(P)}(X)$ exclusive production (the left picture in Fig. 2) with the triple-Pomeron diagram for the SDD of proton at high missing masses (the right picture in Fig. 2), one immediately pays attention to some geometrical likeness between these two diagrams. Indeed, the vertex of two-Pomeron fusion to $f_2^{(P)}(X)$ seems to be related to the triple-Pomeron vertex of SDD.

To establish that relation between $g_{PPf_2}(t_1, t_2)$ and the triple-Pomeron vertex function $g_{3P}(t_1, t_2, t_3)$ we need, first, to consider the expression for the SDD triple-Pomeron interaction amplitude (below we represent it in the form used in [7]),

$$\Gamma_{3P}(s, M_X, \vec{\Delta}_1, \vec{\Delta}_2) \approx \frac{1}{s} \left(i + \tan \frac{\pi (\alpha_P(t_1) - 1)}{2} \right) \left(-i + \tan \frac{\pi (\alpha_P(t_2) - 1)}{2} \right) g_{ppP}(t_1) g_{ppP}(t_2) g_{ppP}(t_3) g_{3P}(t_1, t_2, t_3) \times \pi^3 \alpha'_P(t_1) \alpha'_P(t_2) \alpha'_P(t_3) \left(\frac{1}{\xi} \right)^{\alpha_P(t_1) + \alpha_P(t_2)} \left(\frac{M_X^2}{2s_0} \right)^{\alpha_P(t_3)},$$
(3)

Table 1The parameter values for (5) obtainedvia fitting to the elastic scattering data.

via fitting to the	elastic scattering data.
Parameter	Value
$\alpha_{\rm P}(0) - 1$	0.109
$ au_a$	0.535 GeV ²
$g_{ppP}(0)$	13.8 GeV
ag	0.23 GeV^{-2}

where M_X is the missing mass and $\xi = (M_X^2 - m_p^2)/s$ is the energy fraction lost by the diffractively scattered proton, and, second, to replace the exchange by that Pomeron which carries 4-momentum $\Delta_1 + \Delta_2 ((\Delta_1 + \Delta_2)^2 \equiv t_3)$ by the exchange by that virtual particle f_J of spin J and mass m_J which is related to the Pomeron Regge trajectory. Particularly, such a partial de-Reggeization implies the following replacements:

$$\alpha_{\rm P}(t_3) \to J, \quad \alpha_{\rm P}'(t_3) \to \frac{1}{m_l^2 - t_3}, \quad g_{pp\rm P}(t_3) \to g_{pp\rm P}^{(J)}(t_3), \quad g_{3\rm P}(t_1, t_2, t_3) \to g_{\rm PPf_J}^{(J)}(t_1, t_2, t_3), \tag{4}$$

where $g_{ppP}^{(J)}(t_3)$ is the structure function related to the tensor structure $p_2^{\alpha_1}...p_2^{\alpha_J}$ in the tensor current of the proton carrying 4-momentum p_2 in the initial state, and $g_{3P}^{(J)}(t_1, t_2, t_3)$ is the structure function related to the tensor structure $\Delta_1^{\alpha_1}...\Delta_1^{\alpha_J}$ in the partially de-Reggeized triple-Pomeron vertex (these tensor structures dominate in the kinematic region $M_X \gg 1$ GeV, because $M_X^2 \approx 2(\Delta_1 p_2)$ in that range).

Now it is obvious that $g_{PPf_2}(t_1, t_2) \equiv g_{PPf_2}^{(2)}(t_1, t_2, M_{f_2}^2)$, *i.e.*, it corresponds to the triple-Pomeron vertex function $g_{3P}(t_1, t_2, t_3)$ in the limit $\{t_3 \rightarrow k^2 = M_{f_3}^2, \alpha(t_3) \rightarrow J = 2\}$.

For quantitative predictions we need, first of all, to fix the model degrees of freedom, namely, the unknown functions $\alpha_P(t)$, $g_{ppP}(t)$, and $g_{PPf_2}(t_1, t_2)$. The Pomeron Regge trajectory and the Pomeron coupling to nucleon should be the same as in the *pp* elastic scattering [6]:

$$\alpha_{\rm P}(t) = 1 + \frac{\alpha_{\rm P}(0) - 1}{1 - \frac{t}{\tau_a}}, \quad g_{pp\rm P}(t) = \frac{g_{pp\rm P}(0)}{(1 - a_g t)^2}, \tag{5}$$

where the free parameters take on the values presented in Table 1.

As well, it was argued in [7] that

$$g_{3P}(t_1, t_2, t_3) \approx g_{3P}(0, 0, 0) \approx 0.64 \text{ GeV}$$
 (6)

in the kinematic range relevant for SDD. The main hypothesis we use further is the assumption that this equality may be extended to the $f_2^{(P)}(X)$ CEP region:

$$g_{\text{PPf}_2}(t_1, t_2) \equiv g_{3\text{P}}(t_1, t_2, M_{f_2}^2) \approx g_{3\text{P}}(t_1, t_2, 0) \approx g_{3\text{P}}(0, 0, 0).$$
(7)

Having fitted $\alpha_P(t)$ and $g_{ppP}(t)$ to the elastic scattering data and the value of $g_{3P}(0, 0, 0)$ to the data on the proton SDD, and, next, having made assumption (7), we are able to estimate the bare amplitude (2) of the $f_2^{(P)}(X)$ CEP.

To calculate the corresponding CEP cross-section (1) we need to take account of the multi-Pomeron exchanges between the incoming protons and the outgoing ones.¹ It can be done in the same way as for the proton SDD cross-section [7]. Then, the full helicity amplitude can be approximated by the expression

$$T^{(\lambda)}(s,\xi_{1},\xi_{2},t_{1},t_{2},\phi) \approx T^{(\lambda)}_{bare}(s,\xi_{1},\xi_{2},\vec{\Delta}_{1\perp},\vec{\Delta}_{2\perp}) + \\ + \frac{1}{16\pi^{2}s} \int d^{2}\vec{q}_{1\perp} A(s,-\vec{q}_{1\perp}^{2}) T^{(\lambda)}_{bare}(s,\xi_{1},\xi_{2},\vec{\Delta}_{1\perp}-\vec{q}_{1\perp},\vec{\Delta}_{2\perp}+\vec{q}_{1\perp}) + \\ + \frac{1}{16\pi^{2}s} \int d^{2}\vec{q}_{2\perp} T^{(\lambda)}_{bare}(s,\xi_{1},\xi_{2},\vec{\Delta}_{1\perp}-\vec{q}_{2\perp},\vec{\Delta}_{2\perp}+\vec{q}_{2\perp}) A(s,-\vec{q}_{2\perp}^{2}) + \\ + \frac{1}{(16\pi^{2}s)^{2}} \int d^{2}\vec{q}_{1\perp}d^{2}\vec{q}_{2\perp} A(s,-\vec{q}_{1\perp}^{2}) T^{(\lambda)}_{bare}(s,\xi_{1},\xi_{2},\vec{\Delta}_{1\perp}-\vec{q}_{1\perp}-\vec{q}_{2\perp},\vec{\Delta}_{2\perp}+\vec{q}_{1\perp}+\vec{q}_{2\perp}) A(s,-\vec{q}_{2\perp}^{2}) ,$$

$$(8)$$

where the absorption subamplitudes A(s, t) are computed in the following way:

$$A(s,t) = 4\pi s \int_{0}^{\infty} db^2 J_0(b\sqrt{-t}) \left(e^{i\delta(s,b)} - 1\right),$$

$$\delta(s,b) = \frac{1}{16\pi s} \int_{0}^{\infty} d(-t) J_0(b\sqrt{-t}) \delta_{\rm P}(s,t),$$
(9)

¹ For detailed discussion of the importance and significance of such absorptive corrections in high-energy CEP, see papers [8] and [9] and references therein.



Fig. 3. The model *t*- and ϕ -distributions of the $f_2(1950)$ CEP at $\sqrt{s} = 13$ TeV obtained via integration over the range { $M_{f_2} - \Gamma_{f_2} < \sqrt{k^2} < M_{f_2} + \Gamma_{f_2}$ and $\xi_{1,2} > 10^{-4}$ }.

$$\delta_{\mathrm{P}}(s,t) = \left(i + \tan \frac{\pi \left(\alpha_{\mathrm{P}}(t) - 1\right)}{2}\right) g_{pp\mathrm{P}}^{2}(t) \pi \alpha_{\mathrm{P}}'(t) \left(\frac{s}{2s_{0}}\right)^{\alpha_{\mathrm{P}}(t)}$$

3. Application to the WA102 data and predictions for the CEP of $f_2^{(P)}(X)$ at the LHC

It was shown in [5] that $f_2(1950)$ is the most promising candidate for the status of the light tensor glueball lying on the Pomeron Regge trajectory. If we put $M_{f_2} = 1.944$ GeV and $\Gamma_{f_2} = 472$ MeV [10], then we obtain the following prediction for the $f_2(1950)$ CEP cross-section at $\sqrt{s} = 29.1$ GeV integrated over the range { $M_{f_2} - \Gamma_{f_2} < \sqrt{k^2} < M_{f_2} + \Gamma_{f_2}$ and $\sqrt{|t_{1,2}|} < 1$ GeV}:

$$\sigma_{p+p \to p+f_2(1950)+p}^{model}(29.1 \text{ GeV}) = \sum_{\lambda} \sigma_{p+p \to p+f_2(1950)+p}^{(\lambda)}(29.1 \text{ GeV}) \approx 0.33 \,\mu\text{b} \,.$$
(10)

This estimation is 8.5 times less than the measured experimental value [11]:

$$\sigma_{p+p \to p+f_2(1950)+p}^{WA102}(29.1 \text{ GeV}) = (2.788 \pm 0.175) \,\mu\text{b} \,. \tag{11}$$

Such a divergence is due to the fact that the characteristic value $\xi_{1,2} \sim 0.07$ of the energy fractions lost by protons in the WA102 experiment is so high that the combined contribution of the secondary Reggeon exchanges to the bare amplitude of CEP may be comparable to the double Pomeron exchange term (2) or even may dominate over it. Therefore, the obtained model underestimation of the $f_2(1950)$ production cross-section seems quite natural.

The characteristic value of $\xi_{1,2}$ at the LHC is much lower and the dominance of the two-Pomeron fusion term in the bare amplitude is guaranteed. The model computation of the $f_2(1950)$ CEP cross-section at $\sqrt{s} = 13$ TeV integrated over the kinematic range { $\xi_{1,2} > 10^{-4}$, $\sqrt{|t_{1,2}|} < 1$ GeV, and $M_{f_2} - \Gamma_{f_2} < \sqrt{k^2} < M_{f_2} + \Gamma_{f_2}$ } yields the following value:

$$\sigma_{p+p \to p+f_2(1950)+p}^{(\xi_{1,2}>10^{-4})}(13 \text{ TeV}) \approx 0.22 \,\mu\text{b}\,.$$
(12)

The corresponding model *t*- and ϕ -distributions are presented in Fig. 3.

4. Discussion

The above-considered model is grounded on the fact that the vertex function $g_{PPf_2}(t_1, t_2)$ related to the CEP of the discussed light tensor glueball $f_2^{(P)}(X)$ via two-Pomeron fusion and the triple-Pomeron vertex function $g_{3P}(t_1, t_2, t_3)$ which governs the SDD of proton at high energies and high missing masses are just different branches of the same analytic function. This fact is model-independent.

As well, for calculation of the corresponding CEP cross-sections we assumed the negligibility of the nontrivial analytic structure of function $g_{3P}(x, y, z)$ in the unique kinematic region which covers both the ranges relevant for the reaction $p + p \rightarrow p + f_2^{(P)}(X) + p$ and the high-missing-mass SDD of proton. Having estimated the cross-section of the $f_2^{(P)}(X)$ CEP and its partial widths of decay to pairs of light mesons (see paper [5]), we can try to distinguish this tensor glueball among other vacuum resonances produced exclusively at the RHIC or the LHC and decaying through, say, the $\pi^+\pi^-$ and K^+K^- channels.²

The most weak point of the proposed model is assumption (7) which is an extension of assumption (6) and seems to be very strong. In its turn, assumption (6) is based just on the fact that it allows to obtain an estimation of the SDD cross-section logarithmic *t*-slope

 $^{^2\,}$ In the author's opinion, the most promising candidate is $f_2(1950).$

in the range 0.05 GeV² $\leq -t \leq 0.11$ GeV² at $\sqrt{s} = 1.8$ TeV, $B^{model} = 11.7$ GeV⁻² [7], which is consistent with the value measured by the E-710 Collaboration, $B^{E-710} = (10.5 \pm 1.8)$ GeV⁻² [12]. Other data on the *t*-behavior of the proton SDD cross-section are available only in those kinematic ranges wherein the secondary Reggeon exchange contributions are comparable to the triple-Pomeron term (for details, see [7]). The CMS data on the SDD ξ -distribution at $\sqrt{s} = 7$ TeV and $10^{-5.5} < \xi < 10^{-2.5}$ [13] do not allow to confirm the adequacy of assumption (6) in the relevant kinematic range. The replacement, say, $g_{3P}(0, 0, 0) \rightarrow g_{3P}(0, 0, 0) \cdot e^{a(t_1+t_2+t_3)}$, where a = 0.5 GeV⁻² yields just a slight (5-7%) decrease of $d\sigma \frac{SDD}{d\xi}$ in the above-mentioned ξ -interval, which can be easily compensated by an increase of $g_{3P}(0, 0, 0)$. Thus, at first glance, approximation (6) has a very weak phenomenological foundation even at low negative values of t_1 , t_2 , and t_3 , and, hence, its extension up to $t_3 \rightarrow M_{f_2}^2 \sim 4$ GeV² seems unjustified. However, from the phenomenological standpoint, assumptions (6) and (7) are supported by the established behavior of the Pomeron

However, from the phenomenological standpoint, assumptions (6) and (7) are supported by the established behavior of the Pomeron couplings to various light mesons and photons. It was shown in [5] that approximations $g_{hhP}(t) \approx g_{hhP}(0)$ (here $h = \pi$, K, ρ , ω , ϕ , γ) are not only consistent with available data on the corresponding exclusive reactions of meson-proton and photon–proton scattering, but, being extended to the interval $0 < t < M_{f_2}^2$, allow to obtain the estimations of the quantities $\Gamma_{f_2(1950) \to \gamma\gamma} \Gamma_{f_2(1950) \to K+K^-}/\Gamma_{f_2(1950) \to X}$ and $\Gamma_{f_2(1950) \to \gamma\gamma} \Gamma_{f_2(1950) \to \pi^0 \pi^0}/\Gamma_{f_2(1950) \to X}$, which are quite in agreement with the Belle Collaboration data [14,15]. Hence, we could expect that the function $g_{3P}(t_1, t_2, t_3)$ (the Pomeron coupling to the Pomeron) in the region $\{-1 \text{ GeV}^2 < t_3 < M_{f_2}^2; -1 \text{ GeV}^2 < t_{1,2} < 0\}$ is not an exception to the rule, and, thus, approximation (7) should be kept in mind as a quite probable property of the triple-Pomeron coupling.

Certainly, it is possible that the true value of $g_{3P}(0, 0, M_{f_2}^2)$ is not equal to the value of $g_{3P}(0, 0, 0)$. However, in view of the aforesaid, we could expect, at least, the validity of a much weaker assumption in the range of low negative $t_{1,2}$:

$$g_{\text{PPf}_2}(t_1, t_2) \approx g_{\text{PPf}_2}(0, 0) \equiv g_{3\text{P}}(0, 0, M_{f_2}^2).$$
(13)

In this case, we can not provide a prediction for the $f_2^{(P)}(X)$ CEP cross-section, but the distributions presented in Fig. 3 remain unchanged.

Moreover, relations analogous to (13) could take place in the region of low $|t_{1,2}|$ for the CEP of various *C*-even isoscalar vacuum mesons. Such a variant does not seem exotic, if we consider the *t*-evolution of the ρ -Reggeon and *f*-Reggeon couplings to pion in the region t > 0. Application of formula (A.4) from the Appendix of [5] to the $\pi\pi$ decays of mesons ρ (770), ρ_3 (1690), f_2 (1270), and f_4 (2050) [10] yields the following ratios:

$$\frac{|g_{\pi\pi\rho}(M_{\rho_3(1690)}^2)|}{|g_{\pi\pi\rho}(M_{\rho(770)}^2)|} = 1.1 \pm 0.05, \quad \frac{|g_{\pi\pi f}(M_{f_4(2050)}^2)|}{|g_{\pi\pi f}(M_{f_2(1270)}^2)|} = 0.56 \pm 0.06.$$
(14)

Thus, the couplings of these Reggeons to pion have very weak *t*-dependence in the resonance region. Consequently, we could expect that the dependence of the Pomeron–Pomeron-meson vertex functions (the meson couplings to the Pomeron) on t_1 and t_2 in the range $|t_{1,2}| < 1 \text{ GeV}^2$ might be weak, at least, for some of vacuum mesons.

Then, the distributions over the kinematic ranges relevant for the nucleon–nucleon elastic scattering, SDD of proton, and CEP of some of vacuum resonances (including $f_2^{(P)}(X)$) turn out strongly correlated, since they are determined by the *t*-behavior of $\alpha_P(t)$ and $g_{ppP}(t)$ only, while the corresponding Pomeron–Pomeron–meson vertex functions can be treated just as some constants which determine the values of the meson CEP cross-sections, but have no influence on the form of the distributions over kinematic variables. Therefore, the proposed approach could be very useful in phenomenology of high-energy CEP of light vacuum mesons (see [16] for more details).

In the very end, it should be pointed out that approximations (7) and (13) are still just assumptions, though they have reasonable phenomenological grounds. Besides, these restrictions are so stiff that they can be easily confirmed or discriminated by the forthcoming experimental data on the CEP of light vacuum resonances from the RHIC and the LHC.

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