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Алгебра, теория чисел и дискретная геометрия: современные проблемы, приложения и проблемы истории

Материалы XVI Международной конференции, посвященной 80-летию со дня рождения профессора Мишеля Деза

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АЛГЕБРА, ТЕОРИЯ ЧИСЕЛ И ДИСКРЕТНАЯ ГЕОМЕТРИЯ: СОВРЕМЕННЫЕ ПРОБЛЕМЫ, ПРИЛОЖЕНИЯ И ПРОБЛЕМЫ ИСТОРИИ

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чисел $\mathbb{Z}$ относительно простого числа p. Поле K называется *полем расщепления группы A,* если $A \otimes R \cong D \oplus F$, где $R = K \cap \widehat{\mathbb{Z}}_p$, $\widehat{\mathbb{Z}}_p$ — пополнение $\mathbb{Z}_p$ в p–адической топологии, D — делимый R–модуль, F — свободный R–модуль. Кольцо R в этом случае называется *кольцом расщепления группы A*. Кольцо расщепления, не содержащее собственных колец расщепления группы A, называется *минимальным кольцом расщепления группы A.* Поле расщепления называется *минимальным полем расщепления группы A,* если не содержит собственных полей расщепления группы A.

Теорема 1. *Всякая неразложимая p–локальная группа без кручения A с минимальным квадратичным полем расщепления $K = \mathbb{Q}(\pi)$ изоморфна группе $G = \langle a, b \mid a = \pi b\rangle_* \subset \widehat{\mathbb{Z}}_p b$, где π — примитивный элемент расширения $\mathbb{Q}(\pi) = K$.*

Следствие 1. *[1] Всякая сильно неразложимая p–локальная группа без кручения A с минимальным квадратичным полем расщепления $K = \mathbb{Q}(\pi)$ изоморфна группе*

$$G = \langle a, b \mid a = \pi b\rangle_* \subset \widehat{\mathbb{Z}}_p b.$$

Следствие 2. *Всякая неразложимая (сильно неразложимая) p–локальная группа без кручения A с минимальным квадратичным полем расщепления $K = \mathbb{Q}(\pi)$ изоморфна аддитивной группе минимального кольца расщепления $R = K \cap \widehat{\mathbb{Z}}_p$.*

Следствие 3. *Если кольца эндоморфизмов $\mathrm{E}(A)$ и $\mathrm{E}(B)$ неразложимых p–локальных групп без кручения A и B с минимальным квадратичным полем расщепления K изоморфны, то группы A и B также изоморфны.*

Следствие 4. *В классе неразложимых p–локальных групп без кручения с минимальным квадратичным полем расщепления K всякая группа определяется с точностью до изоморфизма минимальным кольцом расщепления.*

Следствие 5. *В классе неразложимых p–локальных групп без кручения с минимальным квадратичным полем расщепления K всякая группа определяется с точностью до изоморфизма аддитивной группой минимального кольца расщепления.*

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On the classification of Abelian and non-Abelian groups of the unit n-arity complex / hypercomplex numbers

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О классификации Абелевых и неабелевых групп единичных n-арити комплексных / гиперкомплексных чисел

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During the last 20 years we related a development to the searching for new Riemannian and tensor structures in multidimensional spaces $D \geq 5$ based on the theories of new hyper-numbers, new algebras and new symmetries. We refer to the theory of numbers - theory of reflexive numbers and then to theory of the cyclic C^n- complex numbers[1, 2].

The geometrical nature of Abelian group we could search for in the theory of Abelian $n-ary$ complex numbers. We took the idea of considering $n-ary$ complex numbers in Euclidean R^n-spaces as a tool for finding new symmetries in connection with the Calabi-Yau classification of spaces of any dimension $CY(d)$, $d = complex.dim. = 2p - real.dim.$, which we made on the basis of the $n-ary$ theory of reflexive projective numbers [3, 4, 5]. d-dimensional Calabi-Yau spaces are multi-dimensional generalizations of the $d=1$ one-dimensional torus This classification allowed us to see new n-ary structures in Newtonian polyhedrons for Calabi-Yau spaces $CY(d)$ with the holonomy group $SU(p)$, $d = 3,4,5,\ldots$ One of the possible ways to establish a connection between external and internal symmetries is to cover the groups of internal symmetries of groups of external space-time symmetries by groups? So we already know such examples with double covering: $SU(2) \approx S^3/Z_2 \approx SO(3) \approx \mathbb{RP}^3$, $U(1)_{EM}$, $SU(2)$ -spin, $SL(2,C)$ -matter-antimatter, $N^{O\pm} = 3$ with the group $SU(3^{O\pm})$ -color symmetry, $N_g = 3$-number of generations of quark-leptons.

According to the Abelian C_n- cyclic group complexification of the Euclidean $\mathbb{R}^n$ spaces followed to this method we consequently constructed the series-$n = 3,4,5,6;12$ of the n-dimensional $(n-1)$-parameter Abelian group- hypersurfaces with $n = 2,3,\ldots$. We determine the Abelian group symmetries for such spaces. The n-ary complex numbers lead to two isomorphic, n-ary "unitary"and "orthogonal Abelian (n-1) -parametric symmetry groups, which could be the basis for describing the invisible light of the universe. The feature of the our approach is the appearance of noncompact Abelian symmetry groups.

The theory of Abelian complex numbers is based on the complexification of the Euclidean $\mathbb{R}^n$-space[5]–[6]

$$z = x_0 q_0 + x_1 q + ... + x_{(n-1)} q^{(n-1)}, \tag{1}$$

using $C_n = q_0, q, ..., q^{(n-1)} : q^n = \pm q_0, q_0 - unit$ - cyclic groups of their n-one-dimensional irreducible representations for conjugation operations

$$\tilde{q} = q^{\{1\}} = jq,\ \tilde{\tilde{q}} = q^{\{2\}} = j^2 q,\ ..., q^{\{n-1\}} = j^{(n-1)} q; \qquad q^{\{n\}} = q; \qquad j = e^{(2\pi i/n)} \tag{2}$$

which allow to determine the norm $||z||^n = z \cdot z^{\{1\}} \cdot \ldots \cdot z^{\{n-1\}}$, which has composite group properties $||z_1 \cdot z_2||^n = ||z_1||^n \cdot ||z_2||^n$, which allows for n-ary complex numbers with a single norm to determine (n -1) - parametric Abelian groups. Following the Abelian C_n-complexification of Euclidean spaces R^n, we successively construct the series of Abelian $(n-1)$ -parameter-invariant hypersurfaces $||z||^n = F_0(x_0, ..., x_{(n-1)}) = 1$ for $n = 3,4,5,6,...,12$ (further expansion is obvious) we study the process of, we derive Euler formulas as the basis for the derivation of n-ary-unitary Abelian groups in the $(n \times n)$ -matrix representation. For illustration we present the expressions of algebraic equations for the hypersurfaces defined by the C^n, - cyclic unit numbers only for $n \leq 6$ for both cases, A) $q^n = q_0$, B) $q^n = -q_0$,respectively (these cases can be linked by extended Wick twist):

Generalizations of the n-dimensional trigonometry and the Pythagorean theorems for n-dimensional simplexes in each n-ary case have been got, for example, let see $n = 3$:

$$e^{(q\alpha+q^2\beta)} = c_0(\alpha,\beta)q_0 + s_0(\alpha,\beta)q + t_0(\alpha,\beta)q^2, \tag{3}$$

where $c_0^3 + s_0^3 + t_0^3 - 3c_0s_0t_0 = 1$. The two parameter Abelian ternary "unitarity"is:

$$U = e^{(\alpha q+\beta q^2)},\ U^+ = e^{(j\alpha q+j^2\beta)q^2};\ U^{++} = e^{(j^2\alpha q+j\beta)q^2)}: \qquad U\cdot U^+\cdot U^{++} = \hat{1} \tag{4}$$

Or in matrix form

$$U = \begin{pmatrix} a & qb & q^2c \\ cq^2 & a & qb \\ qb & cq^2 & a \end{pmatrix}, U^+ = \begin{pmatrix} a & jqb & j^2q^2c \\ j^2cq^2 & a & jqb \\ jqb & j^2cq^2 & a \end{pmatrix}, U^{++} = \begin{pmatrix} a & j^2qb & jq^2c \\ jcq^2 & a & j^2qb \\ j^2qb & jcq^2 & a \end{pmatrix} \tag{5}$$

$$U\cdot U^+\cdot U^{++} = detU = (a^3+b^3+c^3-3abc)\cdot\hat{1} = \hat{1}. \tag{6}$$

Subsequent consideration in $CN^{(n)}$ - n-ary complexified $\mathbf{R}^n$- Euclidean space the ways of the holomorphizm (and polymorphizm) for the functions:

$$F(z,z^{\{1\}},...,z^{\{n-1\}}) = F_0(x_0,...,x_{(n-1)})q_0 + F_1(x_0,...,x_{(n-1)})q + ... + F_{(n-1)}(x_0,...,x_{(n-1)}q^{\{n-1\}} \tag{7}$$

($z^{\{p\}} - p = 1,2,...,(n-1)$ is the number of the conjugation operations of the n-ary complex number z.) allows us to derive n-dimensional wave equations (of the Laplace / Dirac type) for the harmonic functions $F_a(x_0,...,x_{(n-1)})$ and the corresponding n-spinors, invariant relativity of the corresponding $(n-1)$ -parametric Abelian symmetry groups (for example, see the case n=6 $q^6 = q_0$[4]–[7]:

$$\begin{aligned} &\{[(\partial_0^3+\partial_2^3+\partial_4^3-3\partial_0\partial_2\partial_4) - 3(\partial_0(\partial_3^2-\partial_1\partial_5)+\partial_2(\partial_5^2-\partial_1\partial_3)+\partial_4(\partial_1^2-\partial_3\partial_5))]^2 \\ - &[(\partial_1^3+\partial_3^3+\partial_5^3-3\partial_1\partial_3\partial_5) - 3(\partial_1(\partial_4^2-\partial_0\partial_2)+\partial_3(\partial_0^2-\partial_2\partial_4)+\partial_5(\partial_2^2-\partial_0\partial_4))]^2\}\cdot \\ &\cdot F_a(x_0,\ldots,x_5) = 0 \end{aligned} \tag{8}$$

$a = 0,\ldots,5$ For comarison see at the hypersurface equation for this case $q^6 = q_0$':

$$\begin{aligned} &\{[(x_0^3+x_2^3+x_4^3-3x_0x_2x_4)] - 3[x_0(x_3^2-x_1x_5)+x_2(x_5^2-x_1x_3)+x_4(x_1^2-x_3x_5)]\}^2 - \\ &-\{[(x_1^3+x_3^3+x_5^3-3x_1x_3x_5]+3[x_1(x_4^2-x_0x_2)+x_3(x_0^2-x_2x_4)+x_5(x_2^2-x_0x_4)]\}^2 \\ &= \{F_1^3\}^3 - \{F_2^3\}^2 = 1 \end{aligned} \tag{9}$$

The main conclusion of our calculations is that the Abelian properties of the cyclic groups $C_n$lead to the factorization of all the corresponding group $(n-1)$ -dimensional hypersurfaces $(n-1)$ with the definition of the Abelian group $U^{(n-1)}$, and by analyzing $U^{(n-1)}$ - the invariant Laplace differential equations for arity-n harmonic functions $F_i(x_0,...,x_{(n-1)}), i = 0,...,n-1,$, where the harmonic functions are determined through the decomposition of the holomorphic function [1, 4].

We identified and investigated all Abelian symmetries for the cases $(n = 3,...,12)$, which could serve as opportunities to consider Abelian theories of invisible light of the Universe, interacting with some invisible matter- exotic n-spinor "maarkrions". In this view, we naturally extended the constructions of Abelian n-ary complex numbers studied to the construction of the non-Abelian n-ary hyper-complex numbers starting from two introducing hyper-complex numbers producing the (n^2-1) (for $tsu(3)$- 8) generators[1]–[6]:

$$Z = x_0Q_0 + x_1Q_1 + ... + x_{(n^2-1)}Q_{(n^2-1)} : (Q_k)^n = \pm Q_0; k = 1, ..., n^2-1 \tag{10}$$

The Laplace differential equations can be found from the Cauchy-Riemann equations of order deg= n. Following to Dirac procedure one can get from the differential equations for $n-ary$ harmonic functions the linear differential invariant equations for $n-spinors$:

$$\hat{F}(\partial x_0, ..., \partial x_{n-1})\psi = 0, \qquad \psi = \begin{pmatrix} \eta_0 \\ .... \\ \eta_{n-1} \end{pmatrix} \tag{11}$$

As example we give the commutations relations for $tsu(3)$-algebra:

$$[Q_1, Q_2] = (j^2-j)Q_6; [Q_2, Q_3] = (j^2-j)Q_4; [Q_3, Q_1] = (j^2-j)Q_5 \tag{12}$$

$$\text{etc.: } [Q_k, Q_l] = \pm(j^2-j)Q_m \text{ and } [Q_1, Q_4] = 0; [Q_2, Q_5] = 0; [Q_3, Q_6] = 0. \tag{13}$$

The unit ternary-hypercomplex numbers produce the non-Abelian ternary-unitary STU(3)-group Lee. Using the unusual comutations rules we constructed the hypersurface define by the unit ternary

$$z = (x_0q_0 + x_7q + x_8q^2) + (x_1q_0 + x_2q + x_3q^2)q_1 + (x_4q_0 + x_5q + x_6q^2)q_1^2 \tag{14}$$

The 8 imaginary units $\{Q_k|q, q^2, q_1, qq_1, q^2q_1, q_1^2, qq_1^2, q_1^2, q^2|Q_k^3 = q_0,\}$ produce $tsu(3)$. The ternary hypercomplex units $|z \cdot \tilde{z} \cdot \tilde{\tilde{z}}| = 1$ produce the ternary nonAbelian $TSU(3)$ group Lee respectively.

The corresponding $TSU(3)$-invariant hypersurface $|z \cdot \tilde{z} \cdot \tilde{\tilde{z}}| = 1$ takes the following form [5]–[7]:

$$F(x_0, ..., x_8) = |z_0|^3 + |z_1|^3 + |z_2|^3 - (z_0\tilde{z}_1\tilde{\tilde{z}}_2) - (\tilde{z}_0\tilde{\tilde{z}}_1z_2) - (\tilde{\tilde{z}}_0\tilde{z}_1z_2) = 1 \text{ where} \tag{15}$$

$$z_0 = x_0q_0 + x_7q + x_8q^2 z_1 = x_1q_0 + x_2q + x_3q^2 z_2 = x_4q_0 + x_5q + x_6q^2 \text{ or} \tag{16}$$

$$\begin{aligned} F(x_0, ..., x_8) = x_0^3 + x_7^3 + x_8^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 - 3x_0x_7x_8 - 3x_1x_2x_3 - 3x_0x_7x_8 \\ -3x_0(x_1x_4 + x_2x_5 + x_3x_6) - 3x_7(x_1x_5 + x_2x_6 + x_3x_4) - 3x_8(x_1x_6 + x_2x_4 + x_3x_5) = 1 \end{aligned} \tag{17}$$

That is, the result of correctness was the vanishing of all contributions V_k in the decomposition

$$V = F(x_0, x_1, ..., x_8)Q_0 + V_1(x_0, x_1, ..., x_8)Q_1 + ... + V_8(x_0, x_1, ..., x_8)Q_8, \tag{18}$$

that is, out of the possible 729 terms, only 45 terms remained non-zero! Thus, the constructed generators $Q_a; a = 1, ...,$ have more complex than binary quaternions, the commutation relations $Q_aQ_b = j^kQ_bQ_a; j = e^{2\pi i/3}$, where the value of $k = 0, 1, 2$ depends on the choice of the generator subgroup (there are three $\{Q_1, Q_2, Q_3\}$,$\{Q_4, Q_5, Q_6\}$,$\{Q_7, Q_8\}$, and they form the algebra tsu (3). The hypersurface itself is a group manifold defined by the ternary group $TSU(3)$. This group and its algebra are fundamentally different from the Cartan-Lie group of $SU(3)$ and its algebra $su(3)$ defined by the 8 Gell-Mann generators. hand drawing)

The corresponding cubic Maarkri-hypersurface takes in $\mathbb{R}^8$ the following form [7]–[9]:

$$U = \begin{pmatrix} z_0 & qz_1 & q^2z_2 \\ q^2\tilde{z}_2 & \tilde{z}_0 & q\tilde{z}_2 \\ q\tilde{\tilde{z}}_1 & q^2\tilde{\tilde{z}}_2 & \tilde{\tilde{z}}_0 \end{pmatrix}, \tag{19}$$

$$DetU = x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + x_8^3 - 3x_0x_7x_8 - 3x_1x_2x_3 - 3x_4x_5x_6 \qquad (20)$$
$$-3x_0(x_1x_4 + x_-2x_5 + x_3x_6) - 3x_7(x_1x_5 + x_2x_6 + x_3x_4) - 3x_8(x_1x_6 + x_2x_4 + x_3x_5) = 1$$

The new commutation rules applied in quantum field theory can lead to the adoration of the Pauli principle, which states the following that could exist in one level the $n = 3, 4, 5$ identical particles.

Using the unusual Maarkri rules of anti-commutation we constructed the norm-division algebra for hyper-ternary complex numbers[8, 9]:

$$\{q_a, q_0 = \hat{1} | q_a^3 = q_0; a = 1, ..., 8\}. \qquad q_a q_b = j q_b q_a, \qquad or \qquad q_a q_b = j^2 \qquad a \neq b \qquad (21)$$

Similarly, considering n = 4.5.6, ... ary hyper-complex numbers, one can construct n-ary algebras and n-ary groups with corresponding commutation relations, where $j = e^{(2\pi i/n)}$.

These compositions relations generalize the well-known anticommutation relations between imaginary quaternions $\{e_1, e_2, e_3\}$, the matrix realization of which are the Pauli matrices

$$e_1 = i\sigma_1, e_2 = i\sigma_2, e_3 = -i\sigma_3 : e_\alpha e_\beta = -e_\beta e_\alpha, \ \alpha \neq \beta.$$

An important property of the ternary (n-ary) group $TSU(3)$ in the matrix formalism is the introduction of new concepts of Complex conjugation, Transposition, Hermitian conjugation and Unitarity: $UU^+U^{++} = 1$. The next step was connected with the further study of ternary Clifford algebras in their relationship with binary Clifford algebras, which would allow building n-spinors material matter with unusual quantum properties.

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