XXXI International Workshop on High Energy Physics:
Critical Points in the Modern Particle Physics
International Journal of Modern Physics: Conference Series
Vol. 47 (2018) 1860089 (5 pages)

DOI: 10.1142/S2010194518600893

Cosmology of Bigravity

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Published 6 June 2018

This article discusses cosmology, bimetric gravity and their possible interplay.

Keywords: Massive Gravity; Bimetric Gravity; Alternative Theories; Universe.

1. Introduction

Recently we celebrated one hundred years of General Relativity (GR) — the theory that had radically changed our vision of the world and especially of its large scale structure. It was created in Germany in 1915 after great efforts by Albert Einstein supported by Marcel Grossmann and David Hilbert. The second step was done in Russia in 1922 by Alexander Friedmann[1] who had uncovered a simple dynamical equation for the geometry of the Universe, now called after him. The history of relativistic cosmology is rather dramatic, see, for example Ref. 2, its heroes are also Lemetre, Eddington, Milne, Gamow, Hoyle and many others. At last now the cosmology has got a status of an exact fundamental science having maybe more open problems than any other. And the first registrations of gravitational waves promise a lot of new surprises in the near future.

Bimetric theories of gravity are among the possible ways for further development of the gravitation theory. People said that the first talk in this direction was given in the USSR where Nathan Rosen, a protege by Einstein, worked in 1936 – 1938,[3] the first publication appeared in 1940[4] and later this approach was developed.[5] A great amount of work in bimetric gravity was done by A.A. Logunov and his collaborators.[6] They have concluded that an introduction of the second space-time metric leads to an appearance of the massive graviton. When the second metric is fixed to be Minkowskian the cosmological equations provide limits for the scale factor, both from below, and from above. Therefore a cyclic Universe evolution was predicted. Also the gravitational collapse was determined to be terminated in the vicinity of the Schwarzshild radius. But a number of degrees of freedom for the gravitational field in this theory is more than expected for a massive spin 2 field. As most physicists believe this leads to instability coming from the so called Boulware-Deser ghost.[7] Anyway an opportunity that the massive graviton can exist is taken into account now and limits on its mass are always calculated for any experimental data.[8] The popular model for massive gravity is now the dRGT[9] (de Rham, Gabadadze and Tolley) theory. This theory naturally suggests an explanation for the"dark energy" or the cosmological constant from the appearance in the Friedmann equation some terms having the order of graviton mass squared. An extended version of this theory which contains two dynamical metrics was proposed by Hassan and Rosen[10] and called as bigravity. This article is intended to discuss essential features of the bigravity and its applications to cosmology.

2. Bigravity

The simplest variant of bigravity is a theory with the following Lagrangian

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_M - \frac{2m^2}{\kappa_g} U(f_{\mu\nu}, g_{\mu\nu}), \tag{1}$$

which includes two copies of the Hilbert-Einstein Lagrangian

$$\mathcal{L}_g = \frac{1}{\kappa_g}\sqrt{-g}g^{\mu\nu}R^{(g)}_{\mu\nu}, \qquad \mathcal{L}_f = \frac{1}{\kappa_f}\sqrt{-f}f^{\mu\nu}R^{(f)}_{\mu\nu}, \tag{2}$$

the dRGT potential U, and the matter Lagrangian $\mathcal{L}_M$. Here κ_g and κ_f are two gravitational constants, m is the graviton mass. Potential $U(f_{\mu\nu}, g_{\mu\nu})$ in the general case is an ultralocal non-polynomial function of the two metric tensors. It is formed as a linear combination of the symmetric polynomials e_i of the matrix square root $\mathsf{X}^\mu_\nu = \left(\sqrt{g^{-1}f}\right)^\mu_\nu$

$$U = \sqrt{-g}\sum_{n=0}^{4}\beta_n e_n(\mathsf{X}) = \beta_0\sqrt{-g} + \ldots + \beta_4\sqrt{-f} \equiv N\tilde{U},$$

where λ_i are eigenvalues of X, and N is the lapse function,

$$\begin{aligned}
e_0 &= 1,\\
e_1 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,\\
e_2 &= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \lambda_4\lambda_1 + \lambda_1\lambda_3 + \lambda_2\lambda_4,\\
e_3 &= \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_4,\\
e_4 &= \lambda_1\lambda_2\lambda_3\lambda_4,
\end{aligned} \tag{3}$$

The explicit calculation of derivatives $\frac{\partial U}{\partial g_{\mu\nu}}$ and $\frac{\partial U}{\partial f_{\mu\nu}}$ is cumbersome. As a result, articles devoted to the Hamiltonian analysis of the constraint structure in bigravity or in massive gravity are looking complicated and non-transparent. But in dealing with a homogeneous isotropic flat space cosmology one can limit himself to diagonal metrics of the form

$$f_{\mu\nu} = (-N^2(t), \omega^2(t)\delta_{ij}), \qquad \sqrt{-f} = N\omega^3, \tag{4}$$

$$g_{\mu\nu} = (-N^2(t)u^2(t), \xi^2(t)\delta_{ij}), \qquad \sqrt{-g} = Nu\xi^3, \tag{5}$$

and then the dRGT potential occurs a polynomial

$$U = \sqrt{-g}\sum_{i=0}^{4}\beta_i e_i = Nu\xi^3\sum_{i=0}^{4}\beta_i e_i = N(uV + W), \tag{6}$$

where we have introduced a new variable $r = \omega/\xi$ and

$$V = \beta_0\xi^3 + 3\beta_1\xi^2\omega + 3\beta_2\xi\omega^2 + \beta_3\omega^3 \equiv \xi^3 B_0(r), \tag{7}$$

$$W = \beta_1\xi^3 + 3\beta_2\xi^2\omega + 3\beta_3\xi\omega^2 + \beta_4\omega^3 \equiv \xi^3 B_1(r), \tag{8}$$

$$B_i(r) = \beta_i + 3\beta_{i+1}r + 3\beta_{i+2}r^2 + \beta_{i+3}r^3. \tag{9}$$

In fact, the cosmological problem is treated here in the mini-superspace[11] with an ideal fluid as a matter source. The new features in comparison to the standard GR treatment appear to be first a couple of the Friedmann-like equations,

$$H_f^2 = \frac{\Lambda_f}{3}, \tag{10}$$

$$H_g^2 = \frac{\kappa_g}{6}\rho + \frac{\Lambda_g}{3}, \tag{11}$$

where $H_f = \dot{\omega}/N\omega$, $H_g = \dot{\xi}/Nu\xi$, and second a new constraint

$$\Omega = \frac{6m^2}{\kappa_g}\left(\omega H_f - \xi H_g\right)\left(\beta_1\xi^2 + 2\beta_2\xi\omega + \beta_3\omega^2\right) = 0. \tag{12}$$

This constraint together with one of the Friedmann-like equations occur a couple of second class constraints necessary to avoid of the Boulware-Deser ghost. This amount of constraints creates a problem: how to choose independent variables? It seems reasonable to treat as such a variable the ratio of the two scale factors r. Then the Universe evolution may be considered as an evolution of r according to the following equation, where we use some new notations $D_1 = \beta_1 + 2\beta_2 r + \beta_3 r^2$, $\mu = \kappa_f/\kappa_g$, $w = p/\rho$,

$$\dot{r} = N\frac{(1+w)\left(\frac{\mu B_1}{r} - B_0\right)\sqrt{\mu\frac{m^2}{3}\frac{B_1(r)}{r}}}{(\frac{1}{3}+w)\frac{\mu B_1}{r} - (1+w)B_0 + \left(r + \frac{\mu}{r}\right)D_1}, \tag{13}$$

from one critical value to the next critical value.[12] This is due to the fact that the standard (observable) variables such as the energy density of matter or of vacuum become some explicit functions of unobservable variable r:

$$\Lambda_f = \mu m^2\frac{B_1(r)}{r^3}, \tag{14}$$

$$\Lambda_g = m^2 B_0(r), \tag{15}$$

$$\rho = \frac{2m^2}{\kappa_g}\left(\mu\frac{B_1(r)}{r} - B_0(r)\right). \tag{16}$$

Of course, these simple equations do not pretend to take into account effects of quantum gravity and so their domain of validity should be cut at the large values of ρ and Λ. It seems promising to study different choices of the bigravity parameters β_i and κ_f in order to get a best fit of the observable data.

Whereas the evolution of the early Universe is supposed to be known starting from the age of about 1 sec or less, the evolution of the "dark energy" is allowed at the large energy scales and does not contradict to the observable data. Therefore this can be modelled by making different choices of the bigravity free parameters. Also the evolution of "dark matter" energy density may be analysed on the base of the additional proposals. Some of them were made in articles.[13–15]

3. Conclusion

Of course, there are other possibilities of describing cosmology in the frame of the bigravity theory which we are unable to discuss here. After all we remind the works where the special kind of matter coupling only to the second metric $f_{\mu\nu}$ is introduced.[16] Differently, one can also suppose that the matter is coupling to the third "effective" metric formed by the first and second ones taken together.[17] Also another independent third metric which couples to both $g_{\mu\nu}$ and $f_{\mu\nu}$ may be introduced.[18] The bigravitational approach to cosmology seems a promising field of research.

The author is greatful to the participants of the Workshop for their attention and discussion.

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