Hamiltonian cosmology of bigravity

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The purpose of this talk is to give an introduction both to the Hamiltonian formalism and to the cosmological equations of bigravity. In the Hamiltonian language we provide a study of flat-space cosmology in bigravity and massive gravity constructed mostly with de Rham, Gabadadze, Tolley (dRGT) potential. It is demonstrated that the Hamiltonian methods are powerful not only in proving the absence of the Boulware-Deser ghost, but also in addressing cosmological problems.

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Friedmann cosmology in Hamiltonian form

The Friedmann cosmology of General Relativity is given by a Hamiltonian system with one gravitational variable (the scale factor) a, and one matter field, let be scalar ϕ. The total Hamiltonian is a first class constraint (the celebrated Friedmann equation or "the Differential Equation by God") multiplied by a Lagrange multiplier (the lapse N) due to the time reparametrization invariance

$$\mathrm{H} = Na^3 \left(-\frac{6H^2}{\kappa} + \rho + \frac{2\Lambda}{\kappa} \right).$$

The scalar field may be treated as an ideal fluid with $\mathrm{H_M} = Na^3\rho$, and $\mathrm{L_M} = Na^3 p$. Then we can take the scale factor a and the Hubble constant H as a couple of almost canonical variables and the Poisson brackets are

$$\{a, H\} = -\frac{\kappa}{12a^2}, \qquad \{\phi, \pi_\phi\} = 1.$$

Two of Hamiltonian equations are kinematical ones, and other two are dynamical. In particular, the gravitational equation is as follows

$$\dot{H} = \{H, \mathrm{H}\} = -\frac{\kappa}{4} N(\rho + p),$$

and the scalar dynamical equation is equivalent to the conservation law

$$\dot{\rho} + 3NH(\rho + p) = 0.$$

The bigravity is formed as a sum of two copies of General Relativity accompanied by a potential

$$\mathcal{L} = \frac{1}{\kappa_f}\sqrt{-f}R_f + \frac{1}{\kappa}\sqrt{-g}R + \mathcal{L}_{\mathrm{M}}(g_{\mu\nu}, \phi) - U(f_{\mu\nu}, g_{\mu\nu}),$$

where the dRGT potential is formed by means of the symmetric polynomials $e_n(X)$ of matrix $X^\mu_\nu = \left(\sqrt{g^{-1}f}\right)^\mu_\nu$:

$$U = \frac{2m^2}{\kappa}\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n(X) = \frac{2m^2}{\kappa}\left(\beta_0\sqrt{-g} + \ldots + \beta_4\sqrt{-f}\right).$$

Given the Friedmann ansatz for both metrics $f_{\mu\nu} = \mathrm{diag}(-N^2, r^2\chi^2\delta_{ij})$, $g_{\mu\nu} = \mathrm{diag}(u^2N^2, \chi^2\delta_{ij})$ we obtain

$$X = \sqrt{Y} = \mathrm{diag}\left(+\sqrt{u^{-2}}, +\sqrt{r^2}\delta_{ij}\right) \equiv \mathrm{diag}\left(u^{-1}, r\delta_{ij}\right),$$

where e_i are as follows

$$e_0 = 1, \quad e_1 = \frac{1}{u} + 3r, \quad e_2 = \frac{3r}{u} + 3r^2, \quad e_3 = \frac{3r^2}{u} + r^3, \quad e_4 = \frac{r^3}{u}.$$

In bigravity there are two Lagrange multipliers N, u, so we get two primary constraints, the first one is first class, the second is second class

$$H^2 = \frac{\kappa\rho}{6} + \frac{\Lambda(r)}{3}, \qquad (H_f)^2 = \frac{\Lambda_f(r)}{3},$$

where Λ and Λ_f are cubic polynomials of r and r^{-1} respectively

$$\Lambda = m^2\left(\beta_0 + 3\beta_1 r + 3\beta_2 r^2 + \beta_3 r^3\right), \ \Lambda_f = m^2\frac{\kappa_f}{\kappa}\left(\frac{\beta_1}{r^3} + 3\frac{\beta_2}{r^2} + 3\frac{\beta_3}{r} + \beta_4\right).$$

There is one secondary constraint, it is second class and it is factorized, Lagrange multiplier u is determined from the consistency. Only one factor

$$H_f = r^{-1}H,$$

provides nontrivial cosmology[1], allowing to get $H(r)$, $\rho(r)$, and reducing the Universe evolution to one equation for the "hidden parameter"

$$\dot{r} = Nr(1 - ur)H.$$

References

1. V.O. Soloviev, Hamiltonian cosmology in bigravity and massive gravity, Arxiv:1505.00840.