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The Geometry of the Standard Model*

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We discuss the geometrical image of the Standard Model what can require intensive development of non-trivial extension of $D = 3 + 1$ Minkowski space-time what can lead to the exotic spin-fermion structure. According to the cyclic C^n-complexification of the Euclidean $\mathbb{R}^n$ spaces we consequently construct the infinite series of n-dimensional hypersurfaces with $n = 2, 3, \ldots$ The corresponding hyper-complex analysis for the cyclic-holomorphic functions in $\mathbb{M}C^n$-spaces getting the hyper Cauchi-Riemann/ hyper-Laplace equations for "cyclic-holomorhic/cyclic-harmonic" functions, and the Abelian $(n-1)$ dimensional invariant equations for the n-dimensional spinors. The exotic spin-fermion structure of the dark matter could be the explanation of its non-observability.

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1. Standard Model and new space-time geometrical structure of our Universe

The geometrical basis of the modern quantum field theory sucessufully describing the $U(1)_{EM}$- electrodynamic processes, the $SU(3^c)$-gauge quantum chromodynamics and the electroweak interactions based on the $SU(2)_{WI} \times U(1)_Y$- gauge broken symmetry is our space-time world what can be represented as a homogeneous and isotropic $D = (3 + 1)$-four-dimensional continuum. The symmetry properties of the spatial and temporal continuum describe by the Lorentz-Poincaré groups and its representations and some fundamental discrete symmetries- P,T,C. This space-time continuum can be immersed into much huge comprehensive multidimensional world. The modern experimental data derived from the elementary particle physics and astrophysics allow us to estimate the sizes of the expanding visible part of the continuum $\Lambda_{min} \leq \Lambda \leq \Lambda_{max}$, respectively. For more than 60 years the mathematical formalism of quantum field theory was the main tool of the huge galaxy of physicists published during this period thousands and thousands articles. Thus the formalism of quantum field theory includes the geometric foundation of space-time picture of the "visible "world and the operator-functional methods of describing a matter moving and interacting in this environment. But now some phenomena in physics of elementary particles pose the question the need to expand our notions of space and time?! In this case the first question arises of dimension and signature of a new hypothetical world. In our opinion, now modern science close to understanding to the origin of the visible part of universe defined by a D=(3+1)-dimensional space-time

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continuum, obeying to the laws of absolutism speed of light, and the observable fermion matter of which has the "unified" electromagnetic nature. In articles [2] , [19] it was suggested that only the Dirac fermion matter can satisfy to the laws of absolutism speed of light in Minkowsky ($D = 3 + 1$) space-time. Absence of singularities in such a space-time allows you to enter the gauge invariance in a region, which can connect two kinds of matter: the matter substance and radiation. The substance described by the fundamental fermion fields with spin 1/2[4], G.Weyl,[11], and radiation - gauge fields with spin 1. The question of maintaining gauge invariance may depend on the existence of singularities in this space-time, which can be a source of symmetry breaking. This option is actually a violation of gauge symmetry associated with the existence of space-time singularities at small or large distances. Note that the existence of singularities at small distances can lead to a change of the Riemann metric and, therefore, to a dynamical violation of space-time Lorentz symmetry (see for example, [7]). In the case of $SU(2)_{WI} \times U(1)_Y$ gauge symmetry its violation can be carried out without violation of Lorentz symmetry with P-and C- violations and , accordingly, CP- and the CPT-theorem should be performed . The Lie algebra of Lorentz group $SO(3, 1)$ is isomorphic to the algebra of its double covering $Spin(3, 1) = SL(2C)$-group, the irreducible representations of what can be defined by two integer or semi-integer numbers (μ, ν) of the finite-dimensional representations of the $SU(2)_L \times SU(2)_R$ group. The minimal representations of this group are scalar $(0, 0)$ representation) and the Weyl spinors , left-handed $(1/2, 0)$- and right-handed $(0, 1/2)$-representations , what are related by P -parity operation (and complex conjugation): $x_0 \to x_0$, $\vec{x} \to \ \vec{x}$, $(1/2, 0) \to (0, 1/2)$. To describe the $I_\pm$- charged weak currents and combine them to the EM- currents in Weinberg-Salam model $SU(2)_{WI} \times U(1)_Y$ it was used the ideas of the Heisenberg $SU(2)_I$- isotopic group and the following relation $Q_{EM} = I_3 + Y/2$. As one of the main result of a such model it was predicted the neutral weak interactions what was experimentally confirmed in GARGAMELLE CERN neutrino experiments in 1974 year. In this model the P- violation (C-violation) was constructed by hands taking the left- and right handed fermions in different $SU(2)_{WI}$- representations. For breaking the gauge symmetry in Weinberg- Salam model it was used the mechanism in the internal sector of the model what predicted the existence of a new fundamental scalar particle- Higgs boson. However such a mechanism can be considered as purely phenomenological, since its dynamics is far from clear. Mechanism of the appearance of the masses of gauge bosons and fermions is enough formal and it is not clear its link to structural changes of the space-time. At least , in spite of preliminary of strong indications and a lot of discussions CERN plans to continue these experiments for the future cycle of LHC-collider work with planing to get much more the energy of the proton beams. Fermilab also resumed the work on the improvement of the Tevatron to finally clarify the nature of signals detected at CERN collider at energy of 125 Gev. One can propose that the role of the weak sector of the Standard Model is the way to understand the origin of the visible universe. More suppressed processes going to the CP- violation may be associated with an unknown dynamics at the more smaller distances $\sim 10^{-16-17}$cm. In addressing this issue again we faced with the dilemma of the mechanism of these phenomena: defects of the space-time geometry or/and a new dynamics related to the new interactions ? Obviously the issue is closely related to another important problem - the existence of three quark-lepton families. All experimental information on three family mixing and CP-violation can be encoded into the Cabibbo-Kobajashi-Maskawa (CKM)-matrix parameters which also requires explanation! The 3- family mixing explanation is completely going into the mass origin problem. In the second case one should again to study the problem related to the local gauge symmetry breaking - "Higgsology" or unknown a space-time singularity structure. In the depths of this phenomenology is waiting for us very rich physics what can shed light on the production the visible part of Universe!

2. Towards a new spinor-fermion structure

One can suggest that the CPT-symmetry invariance and Q_{EM} -conservation laws can be the prerogatives solely for the Minkowski R^3_1- space-time where the actions of the Lorentz-Poincaré symmetry invariance and $U(1)_{EM}e$ gauge symmetry are spreading but it would not be logically complete if we do not define the fermion matter that fills the space-time continuum should have a universal property, *i.e.* Dirac half-one fermions[2]. [19],[20],[3]. It means that we can imagine the existence of exotic fermion matter , for example, having another spin $1/n$, $n \geq 3$ and without an electromagnetic (color) charge nature[19]. In this picture our visible Dirac Universe forming a topological cycle could be embedded into Meta - Universe having much more reacher the space-time topology. As a messenger between the cycles we suggested neutrinos with Dirac mass equal to zero.

To construct the spin $1/n$ fermion theories first one should find out the examples of the geometrical spaces having such a spinor structure [19],[20],[22]. Taking into account a possibility to imply the spaces with arbitrary spin structure in formulation of the basic principles of the string theory one could significantly expand the assumption touching the D- dimensional pseudo-Lorentz space, in which the string is moving . We think that in this case the string and superstrings theories could considerably extend the set of predictions for modern physics of elementary particles. In according to such geometrical objects one can search for new symmetries, what we already started to study in the class of n-ary symmetries with corresponding n-ary algebras what already have been discussed in literature, for example, [6],[22], and reference there. Opening of three quark lepton generations with absolutely non-standard properties- mass hierarchy, quark mixing and oscillations $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B^0_d - \bar{B}^0_d$, $B^0_s - \bar{B}^0_s$ mesons going with CP-violation (see the first predictions on the significant$B^0_d - \bar{B}^0_d$ mixing [17] [1, 14]), exceptional properties of three neutrinos tell us that we should start with the ternary expansion of the entire binary version of science. One can believe that a new ternary symmetry could explain the following coincidence:

$$N(Color) = N(Family) = N(dimR^3) = 3. \tag{1}$$

The problem of 3 quark- lepton families is the most amazing mystery of nature facing mankind after the discovery the anti-particle world ! If the opening the antiparticles was predicted by Dirac the discovery the muons in cosmic-rays and later the strange particles was an absolute surprise?! Neutrino in three generation theme is playing the special role[2],[19],[3, 20].

Over the last 40 years experimental science made the tremendous progress since the landmark discoveries of charm J/Ψ meson and neutral currents in neutrino interactions with subsequent discoveries of massive *bottom-*,*top-* quarks, tau- lepton and W^+, W^-, Z- gauge bosons. Currently, on the CERN collider is going a fruitful set of statistical data on multiple production processes of these gauge bosons and top-quarks, on massive meson oscillations $D^0 - \bar{D}^0$, $B^0_d - \bar{B}^0_d$, $B^0_s - \bar{B}^0_s$ and decays with the study CP- and CPT-violation there[17],[14],[12]. These same 40 years have brought a lot of frustration in the theoretical studies. Standard model with two additional generations has accumulated a huge number of ambiguities concerning the deepest foundations of our microworld and universe, some of which go to the theory of special relativity, quantum mechanics, the physics of the atom and the nucleus, in cosmology and gravity. Over these years, there were suggested some interesting and promising directions that have been associated with the hope for a great breakthrough in solving the above problems! Among them one can see the situation with grand unified theories, minimal supersymmetric model, the supergravity models, superstring theories, theory of everything, M-and F-theories etc. Why so great ideas and, correspondingly, great theories have not been successful? Experimental searches for predictions of these theories did not get yet a confirmation. Now we wait for the verdict on the fate of the Higgs boson and new science - ”Higgsology” Soon we will see when the Standard Model will be called by Non-Standard Model that will correspond to reality!

With regard to the quantum mechanics there is still a question about the nature of spin. We have already accepted that the fundamental interacted matter of a visible world could be described by the fermion matter and radiation fields with the quantum spin 1/2 and 1, respectively. Invisible fermion matter could have other magnitudes of the spin. In that case, a new kind of matter could not interact directly with our Dirac matter and actually becomes unobservable. The introduction into a theory of a matter with other spin magnitudes require new space-time continuums with peculiarities different from the $D = (3 + 1)$-Minkowsky spaces and even from the $D > 4$ its extensions with pseudo-Lorentzian symmetry. In (super)string theories it was made the suggestion that strings can exist only in $D = (n - 1, 1)$-pseudo-Lorentzian space-time. As we already know this suggestion very strongly confined the applications of them in searching for new phenomena related to the extra large or infinite dimensions?! The attempts to discovery supersymmetry in the frames of the minimal supersymmetric standard model (MSSM) also did not yet get the experimental confirmation. Nontrivial extension of the Lorentz group and, accordingly, the theory of relativity can give new motives that may be related to issues of the nature of the spin. The fundamental questions of atomic and nuclear physics problems escalated into physics of quarks and leptons. Discovery of the fundamental building blocks of nuclear matter, proton and neutron, and isotopic nuclear forces escalated into the $SU(3)$-color model of the gluon interaction of u-and d-quarks. The outstanding questions in nuclear physics of the degeneracy of the masses of the proton and neutron naturally now is related to the problem of u- and d- quark masses. The problem of mysterious mass triangle proton- neutron-electron can become a problem of quantization masses- existence of a fundamental theory of minimal mass and some symmetrical relation between the masses of the u- and d- quarks and electron? Naturally it follows the question of communication its with a geometric justification of Cabibbo-Pati-Salam quark-lepton universality. Finally, discovery in relativistic quantum mechanics anti-particles put in cosmology the problem of disappearing antimatter in Universe.

The main our question to find the geometrical explanation of 3-family Dirac matter what form our visible universe, which is only a small part of a huge hyper-universe. We cannot find a space-time explanation of the number 3 with $D = (n+1)$-Lorentz group for any $n = 3, 4, 5, ...$ The language of mathematics we have to build a new geometrical space with dimensions greater than 4 and symmetries wider than Lorentz group what allow to exist to exotic fermions with spin $1/n, n = 3, 5, 6, 7, ...$?! In the new space-time continuum we would be ready to construct a new quantum field theory of pra-matter (dark) of which was created our observed quark-lepton matter endowed with half-integer spin, electric and color charges, Newton gravity. Critical issues such as spin, charge, color, mass led us to search for new symmetries and geometric spaces, the basics of which we are looking for in the new number theory, algebra, finite groups. If successful, we can understand the mechanism of the formation of our universe.

We can give a little resume. In the late of the 20th century the experimental physics has provided new facts and phenomena that require expansion some basic postulates of quantum physics. Problems of general theory of interaction, the first moments of the birth of the universe around us, appearing in her dark matter and dark energy, the apparent absence of a commensurate amount of antimatter, origin of the quarks -leptons EM- charge structure , the presence of three types of neutrinos and their ”oscillations” indicate that the structure of the world could be much more diverse from the visible world picture. We understand that today a clear theory of neutrino does not exist yet. We do not even know what its space-time wave structure- Dirac or Majorano or something else[2, 3, 19, 20]? Namely the space-time properties of the fundamental fields give us information about the geometrical peculiarities of our space-time! The $D = (3+1)$- Lorentz geometry allows by not contradictory manner could include in the description of all the Dirac fermions, but for neutrinos remains an open question?! In the visible world we were able to observe only partial manifestations of it through the weak interactions. We cannot even claim that we correctly write the wave equation for neutrino?! The special properties of neutrinos could suggest it herald a multidimensional (D = 6 or more) Hyper-universe from which our visible universe was formed and which binds us together, presumably, as we know now, only neutrinos (3x2-dimensional space-time properties?) [2, 19], [20], [3]. In this way you can open the pra-matter and a new types of radiation, to understand the origin of electromagnetic charge matter. Recently for solving the neutrino problems there have been suggested the possible extensions of the $4 = (3+1)$-dimensional space-time by adding one or two additional non-compact large dimensions. The one basic idea of such possible approach has been associated to the proposal that neutrino due its exceptional sterile properties can penetrate into the extended space-time with one or two extra dimensions $D = (4+1)$ or $D = (4+2)$. Of all particles only neutrinos (6 -dimensional new quantum field) claims to be the messenger of the invisible "light" comprehensive hyper-world to our visible light universe . The second idea was initiated by the contrast between the spatial and temporal properties of neutrinos from one side, and quarks/ charged leptons from the other side, what leaves a room to consider the observed three neutrino states as a quantum 6-spinor field in the 6-dimensional space-time, and, in accordance to the "ternary complexity" to consider three implementations of neutrinos as the "particle-" - "anti-particle-" and "anti-anti-particle"- states, in analogy with the 4-dimensional Dirac theory of the electron-positron. The third idea has been based on the non-trivial $D > 4$ -extension of Lorentz-metrics what could conserve the validity all principles of the $D = (3+1)$-special theory of relativity for electromagnetic- charged matter. As a result, it was proposed that the problems of neutrino physics is clearly directed to the search for the new multi-dimensional geometric structures and symmetries.

Existence of a new vacuum says that we need to find a generalization of the Lorentz group, that is, to go beyond the usual geometry of 4-dimensional space-time. The main conclusion is our quark-lepton matter forms our visible universe, which is only a small part of a huge hyper-universe. The language of mathematics we have to build a new geometrical space with dimensions greater than 4 and symmetries wider than Lorentz group. in this new space-time continuum we would be required to construct a new quantum field theory pra-matter of which was created by our observed quark-lepton matter endowed with half-integer spin, electric and color charges gravity. Critical issues such as spin, charge, color, weight led us to search for new symmetries and geometric spaces, the basics of which we are looking for in the new number theory, algebra, finite groups. If successful, we can understand the mechanism of the formation of our universe.

3. Towards an infinite series of C_n hypersurfaces and C_n-holomorphic analysis

Similarly to the cases $n = 3$ [10]we did the same analysis till the C_n cyclic complex numbers till $n = 6$ but a regularity with increasing dimension becomes already clear. The main conclusion of our calculations is that the Abelian specific properties of the finite C_n group lead to the factorization of all the n-dimensional hypersurface defined by the C^n, - cyclic unit numbers for each n with determination the $(n-1)$- parameter Abelian group. This group can be written in the C_n-cyclic Herrmitianl $n \times n$- matrix form with determinant equal to unit. For each case n one can develop the n-gonomy analysis with determination of the corresponding Pythagoras theorem for the n-dimensional simplexes. The number of monomials in the expressions for such hypersurfaces is growing very quickly with n and to study such expressions it is very difficult by standard way $\sim n^n$. We check these ideas by our calculations untill $n = 12$. The effect of factorization gives us the complete understanding of the structure and symmetry of such hypersurfaces. For all cases it is completely clear the forms of C^n cyclic n-dimensional Caushi-Riemann-, Laplace -, and linear equations for the C^n holomorphic-, harmonic- functions and n-spinors, respectively. The forms of Pythagoras theorem in each $\mathbb{R}^n$ and n-gonic analysis can be done also for each n. Since the material what we could has very big volume in this article we present the expressions of algebraic equations for the hypersurfaces defined by the C^n, - cyclic unit numbers only for $n \leq 6$ for both cases, A) $q^n = q_0$, B) $q^n = -q_0$,respectively (these cases can be linked by "extended Wick twist"):

q_n^n	$\tilde{q}_n = e^{(2\pi i/n)} q$	$\langle z z^1 ... z^{n-1} \rangle = R_n^n$
$i^2 = -1$	$\bar{i} = -i$	$x_0^2 + x_1^2 = R^2$
$q_2^2 = 1$	$\tilde{q}_2 = -q_2$	$x_0^2 - x_1^2 = H^2$
$q_3^3 = 1$	$\tilde{q}_3 = e^{(2\pi i/3)} q_3$	$x_0^3 + x_1^3 + x_3^3 - 3x_0x_1x_2 = R^3 = \rho \cdot r^2$
$q_4^4 = 1$	$\tilde{q}_4 = e^{(\pi i/2)} q_4$	$[(x_0 - x_2)^2 + (x_1 - x_3)^2] \cdot [(x_0 + x_2)^2 - (x_1 + x_3)^2]$ $= H_1^2 \cdot H_2^2$
		$[x_3^2 + x_1^2 - 2x_0x_2]^2 - [x_0^2 + x_2^2 - 2x_1x_3]^2 = R_1^4 - R_2^4$
$q_4^4 = -1$	$\tilde{q}_4 = e^{(\pi i/2)} q_4$	$[x_3^2 - x_1^2 + 2x_0x_2]^2 + [x_0^2 - x_2^2 + 2x_1x_3]^2$ $= R_1^4 + R_2^4 = 1$
$q_5^5 = 1$	$\tilde{q}_5 = e^{(2\pi i/5)} q_5$	$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5 - 5x_0x_1x_2x_3x_4-$ $-5\{x_0^3(x_1x_4 + x_2x_3) + x_1^3(x_0x_2 + x_3x_4) + x_2^3(x_1x_3 + x_0x_4)$ $+x_3^3(x_1x_3 + x_0x_4) + x_4^5(x_0x_3 + x_1x_2)\}$ $+5\{x_0(x_1^2x_4^2 + x_2^2x_3^2) + x_1(x_0^2x_2^2 + x_3^2x_4^2)$ $+x_2(x_1^2x_3^2 + x_4^2x_0^2) + x_3(x_2^2x_4^2 + x_0^2x_1^2) + x_4(x_3^2x_0^2 + x_1^2x_2^2\})$
$q_6^6 = 1$	$\tilde{q}_6 = e^{(\pi i/3)} q_6$	$\cdot[(x_0 + x_3)^3 + (x_1 + x_4)^3 + (x_2 + x_5)^3$ $-3(x_0 + x_3)(x_1 + x_4)(x_2 + x_5)]$ $\cdot[(x_0 - x_3)^3 + (x_1 - x_4)^3 + (x_2 - x_5)^3$ $-3(x_0 - x_3)(x_1 - x_4)(x_2 - x_5)]$ $= [u_0^3 + u_1^3 + u_2^3 - 3u_0u_1u_2] \cdot [v_0^3 + v_1^3 + v_2^3 - 3v_0v_1v_2]$ $= \{F_1\}^3 \cdot \{F_2\}^3 = \rho_1 r_1^2 \cdot \rho_2 r_2^2 = 1$
		$\{[(x_0^3 + x_2^3 + x_4^3 - 3x_0x_2x_4)]$ $-3[x_0(x_3^2 - x_1x_5) + x_2(x_5^2 - x_1x_3) + x_4(x_1^2 - x_3x_5)]\}^2-$ $-\{[(x_1^3 + x_3^3 + x_5^3 - 3x_1x_3x_5]$ $+3[x_1(x_4^2 - x_0x_2) + x_3(x_0^2 - x_2x_4) + x_5(x_2^2 - x_0x_4)]\}^2$ $= \{F_1^3\}^3 - \{F_2^3\}^2 = 1$
$q_6^6 = -1$	$\tilde{q}_6 = e^{(\pi i/3)} q_6$	$\{[(x_0^3 + x_2^3 - x_4^3 + 3x_0x_2x_4)]$ $-3[x_0(x_3^2 - x_2x_4) + x_2(x_5^2 + x_1x_3) - x_4(x_1^2 + x_3x_5)]\}^2+$ $+\{[x_1^3 - x_3^3 + x_5^3 + 3x_1x_3x_5]$ $-3[x_1(x_4^2 - x_0x_2)) - x_3(x_0^2 + x_2x_4) + x_5(x_2^2 + x_0x_4)]\}^2$ $= \{\tilde{F}_1^3\}^2 + \{\tilde{F}_2^3\}^2 = 1$

(2)

The mathematical apparatus of the quantum field theory is based from one side on the theory of the complex numbers, one or several variables, as well as the theory of generalized functions, and on the other hand on the properties of the four-dimensional space-time continuum which are incorporated in the Lorentz -Poincaré group symmetry. The quantum numbers of the considered fields and their interactions must strictly comply with the global and local symmetries of the $D = (3 + 1)$- dimensional Minkowsky space-time continuum. The discovery of new particle-fields and interactions may require expansion theory as due to additional internal and /or external the space-time degrees of freedom. Both of these effects may lead to a revision of some basic concepts in quantum field theory. For example, the discovery ternary kind of symmetry could lead to a generalization of such fundamental conception as the Hermiticity! At least, changing of the geometric properties of considered space-time , such as the new global dimensions or singularities, require the significant modernization of the quantum theory. In modern quark-lepton physics we have to find the geometrical status of the quantum phenomena : 3-colors and 3-generations?! Of course we can not find such a status in the Minkowski space-time. In our ideology 3- color can be linked to the singularities in our space time and 3 -generation including neutrinos - to the global changing of its. 500 years have passed since OPENING imaginary number. To understand and accept that number into the science it was a complicated process. Since the introduction of the new ternary imaginary number has already been more than a hundred years! Interest and understanding of the importance of the discovery of more complex imaginary mathematical object causes difficulties perception of his physical

If the first binary complex was introduced by mathematicians to solve algebraic equations is now to build a logical quantum theory without the theory of a complex number is simply unthinkable. His manifestation we are already seeing in the physical phenomena that are very difficult for a simple interpretation. If the imaginary unit we use to describe the laws of propagation of light, in order to describe the neutrinos wave properties require a more complicated structure of imaginary what we saw on the example of a ternary imaginary unit. The binary imaginarity is substructure of ternary imaginarity.

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