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MESONS AND DIQUARKS IN A DENSE QUARK MEDIUM WITH COLOR SUPERCONDUCTIVITY

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In the framework of the Nambu–Jona-Lasinio model with two quark flavors, we investigate the spectrum of meson and diquark excitations of dense quark matter in the phase with color superconductivity. The color $SU_c(3)$ symmetry is spontaneously broken to $SU_c(2)$ in this phase. But instead of the expected five Goldstone bosons in the mass spectrum, we observe only three, among which two bosons obey the quadratic dispersion law. We find the doublet of light diquark states with the mass $\sim 15\,MeV$ and also the heavy diquark resonance ($SU_c(2)$ singlet) with the mass $\sim 1100\,MeV$. The π- and σ-mesons have the mass $\sim 330\,Mev$ in the phase with color superconductivity. The π-mesons are then stable particles, while the σ-meson is stable only in the chiral limit in which the current quark mass m_0 becomes zero. If $m_0 \neq 0$, then the σ-meson mixes with diquarks in the phase with color superconductivity and becomes a resonance with the width $\sim 30\,MeV$.

Keywords: Nambu–Jona-Lasinio model, color superconductivity, mesons, diquarks, Goldstone bosons

1. Introduction

Investigating the properties of hot and/or dense matter composed of strongly interacting particles is a relevant problem in contemporary physics. Under normal conditions, i.e., at a low temperature T and baryon density n_B, a hadronic phase of this matter is realized in which the chiral symmetry is broken and quarks and gluons are unobservable (the confinement phenomenon). It is generally believed that a matter form of strongly interacting particles at a high temperature must be the quark–gluon plasma whose ground state is characterized by chiral symmetry and the presence of quarks and gluons in the mass spectrum. If the temperature is low, then as pressure increases (both the baryon chemical potential μ_B and the baryon density n_B increase in this case), hadrons become closer until at some critical pressure (the baryon density), they group into separate systems composed of quarks, which are therefore called lumps of quark matter. Strongly interacting matter must therefore exist in the form of quark matter at sufficiently high densities (and at low temperatures). Modern estimates give relative densities ten times higher than the density of standard nuclear matter. The quark matter can therefore be observed in experiments on collision of heavy relativistic ions and may presumably exist in neutron star cores.

We review the properties of dense cold quark matter. In this matter state, as well as in the quark–gluon plasma, both the confinement and the spontaneous breaking of the chiral symmetry are absent.[1] But at sufficiently high values of μ_B, a diquark condensate, which breaks the color $SU_c(3)$ symmetry of strong interactions, may appear in such a medium resulting in a system transition to a state with

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[1] If we restrict the consideration to only these two properties, then the quark–gluon plasma does not differ from the dense quark matter formally speaking. But because the gluon density can be neglected compared with the baryon density at very low temperatures, the matter of strongly interacting particles at small T and high n_B is conventionally called quark matter.

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color superconductivity (CSC) [1]. The mechanism resulting in CSC completely differs from the standard superconductivity in which the creation of Cooper pairs is due to the electron attraction arising because of interactions with crystal lattice ions. In quantum chromodynamics (QCD), the quark–gluon interaction results in the attraction of two quarks in the channel that is a color antitriplet. This attraction results in the quarks near the Fermi surface being grouped into Cooper pairs whose condensate is the main indication of the reconstruction of the system ground state and of the appearance of CSC.

The CSC phenomenon was investigated in the QCD framework in [2] at asymptotically high baryon densities $n_{\rm B}$, where the QCD effective coupling constant is small and perturbation theory calculation methods are applicable. But in the domain of small and intermediate values of the density $n_{\rm B}$ (then $\mu_{\rm B} \lesssim 1500\,\text{MeV}$), which may presumably occur in neutron star cores, the strong coupling constant is large, which makes perturbation theory calculations physically inadequate. In this case, effective field theory methods and, in particular, the Nambu–Jona-Lasinio (NJL) model [3] with four-quark interaction are customarily used to investigate dense quark matter. It was shown in the framework of this model that the lower bound of the CSC phase lies at relatively low values of the chemical potential $\mu_{\rm B} \sim 1000\,\text{MeV}$. Moreover, a modification of neutron star evolutionary processes due to the possible presence of CSC at their cores was considered (see [4]).

In the present paper based on our previous publications [5]–[7], we investigate one-particle excitations of the CSC-phase ground state of the dense quark matter at zero temperature in the framework of the NJL model with quarks of two flavors. We mainly focus on calculating meson and diquark masses. Meson properties have been well studied in the vacuum case, but little is known about their properties in a dense (quark) medium. A possible multiple creation of these particles in collision reactions of heavy relativistic ions and their participation in various physical processes inside neutron stars provide additional motivation for our investigation. Although diquarks transfer the color, they are real particles in the CSC phase of quark matter (the color symmetry is broken in this phase); hence, all that was said about mesons holds for diquarks. Furthermore, diquarks may be relevant for studying baryon properties because diquarks can be considered constituents of baryons.

In our investigations, we use methods of quantum field theory, whose basic principles can be found in [8].

2. Effective action and the thermodynamic potential in the NJL model

2.1. The Lagrangian and the effective action. Models of the NJL type are among the most effective tools for investigating a number of physical processes in the QCD low-energy domain. For example, the physics of light mesons [9]–[11] and diquarks [12] and meson–baryon interactions [13] have been successfully described in the framework of the NJL model. Such models are also interesting because they realize the principle of dynamical (chiral) symmetry breaking, a phenomenon actively studied in theories with four-fermion interaction. In particular, the influence of a magnetic field [14] and that of the space curvature and its nontrivial topology [15] on the dynamical symmetry breaking have been studied.

But the model has disadvantages, among which the main disadvantage is the absence of confinement. This is a drawback, but only when describing processes in the vacuum, i.e., at zero temperature and zero baryon density $n_{\rm B}$. For large T and $n_{\rm B}$, the strongly interacting matter, as mentioned in Sec. 1, is not in the hadronic phase, and quarks and gluons hence become observable particles (confinement is absent). The presence of quarks in the mass spectrum of the NJL-type models is then a natural advantage of the model, not disadvantage, when describing physical phenomena under extreme conditions. In this respect, the model has already been used to consider properties of both normal (without CSC) dense quark matter [11], [16] and the CSC phenomenon in the domain of intermediate baryon densities [4], [17].

We here consider the NJL-type model with quarks of two flavors and two coupling constants describing interactions in the quark–antiquark and in the scalar quark–quark channels in the domain of intermediate baryon densities (our consideration is in the Minkowski space–time),

$$L_q = \bar q(\gamma^\nu i\partial_\nu - m_0 + \mu\gamma^0)q + G[(\bar q q)^2 + (\bar q i\gamma^5\vec\tau q)^2] + H\sum_{A=2,5,7}(\bar q^C i\gamma^5\tau_2\lambda_A q)(\bar q i\gamma^5\tau_2\lambda_A q^C), \tag{1}$$

where $\mu \equiv \mu_{\rm B}/3 > 0$ is the chemical potential of the quark number whose fields $q \equiv q_{i\alpha}$ constitute a doublet w.r.t. the flavor $SU(2)$ group ($i = 1, 2$) and a triplet w.r.t. color ($\alpha = 1, 2, 3$) and are four-component Dirac spinors; $q^C = C\bar q^{\rm t}$ and $\bar q^C = q^{\rm t}C$ are the charge-conjugate spinors (here $C = i\gamma^2\gamma^0$ is the charge conjugation matrix and the symbol t denotes transposition). We also use the notation τ_a and λ_A ($a = 1, 2, 3$, $A = 2, 5, 7$) for the respective Pauli 2×2 matrices in the flavor space and for the skew-symmetric Gell-Mann 3×3 matrices in the color space. The Lagrangian L_q is invariant under the color $SU_{\rm c}(3)$ and baryon $U_{\rm B}(1)$ transformations and also under transformations from the flavor $SU(2)$ group (if $m_0 \neq 0$). (The latter extends to the chiral $SU(2)_{\rm L}\times SU(2)_{\rm R}$ group at $m_0 = 0$.) We note that Lagrangian (1) is charge symmetric ($q \to q^C$, $\bar q \to \bar q^C$) at $\mu = 0$, but this symmetry is broken if the chemical potential is nonzero. In our approach, we assume that temperature is zero and the model parameters in numerical calculations take the values

$$G = 5.86\,{\rm GeV}^{-2}, \qquad \Lambda = 618\,{\rm MeV}, \qquad m_0 = 5.67\,{\rm MeV}, \qquad H = \frac{3G}{4}, \tag{2}$$

where Λ is the cutoff parameter of the three-dimensional momentum space over which we integrate when calculating Feynman diagrams. The values of the parameters G, Λ, and m_0 in (2) result in the well-known values of the constant of the weak pion decay $F_\pi = 92.4\,{\rm MeV}$, of the pion mass $M_\pi = 140\,{\rm MeV}$, and of the chiral quark condensate $\langle\bar q q\rangle = -(245\,{\rm MeV})^3$ in the empty space (the procedure for fixing the parameters of the NJL model is well described, e.g., in [9]). Starting from the modern experimental data, we cannot find the coupling constant H in the diquark channel with a sufficient accuracy. We have therefore chosen the relation between G and H in (2) as in the QCD four-quark vertex function in the one-gluon-exchange approximation [4].

We here consider both the properties of the ground state of the system with Lagrangian (1) and the mass spectrum of its quark, meson, and diquark excitations. For this, we must find the system thermodynamic potential (TDP) Ω and its effective action up to the second order in bosonic fields. We begin with the auxiliary Lagrangian L containing bosonic fields,

$$\begin{aligned} L = \bar q(\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma - m_0 - i\gamma^5\pi_a\tau_a)q - \frac{1}{4G}(\sigma\sigma + \pi_a\pi_a) - \frac{1}{4H}\Delta_A^*\Delta_A - \\ - \frac{\Delta_A^*}{2}(\bar q^C i\gamma^5\tau_2\lambda_A q) - \frac{\Delta_A}{2}(\bar q i\gamma^5\tau_2\lambda_A q^C), \end{aligned} \tag{3}$$

where, as in what follows, the repeated indices $a = 1, 2, 3$ and $A, A' = 2, 5, 7$ imply summation. The Lagrangians L_q and L are equivalent for the bosonic field equations of motion, which imply that

$$\begin{aligned} &\sigma(x) = -2G(\bar q q), \qquad \pi_a(x) = -2G(\bar q i\gamma^5\tau_a q), \\ &\Delta_A(x) = -2H(\bar q^C i\gamma^5\tau_2\lambda_A q), \qquad \Delta_A^*(x) = -2H(\bar q i\gamma^5\tau_2\lambda_A q^C). \end{aligned} \tag{4}$$

Relations (4) imply that the meson fields σ and π_a are real-valued, i.e., $(\sigma(x))^\dagger = \sigma(x)$ and $(\pi_a(x))^\dagger = \pi_a(x)$ (the symbol † denotes Hermitian conjugation), whereas the diquark fields Δ_A are complex quantities and therefore $(\Delta_A(x))^\dagger = \Delta_A^*(x)$. It is obvious that $\Delta_A(x)$ and $\sigma(x)$ are scalars and $\pi_a(x)$ are pseudoscalars.

Finally, $\sigma(x)$ and $\pi_a(x)$ are singlets w.r.t. the color $SU_c(3)$ group, while the fields $\Delta_A(x)$ transform in accordance with the antitriplet representation $\bar{3}_c$ of this group.

Using functional methods, we can find the TDP Ω and the effective action $\mathcal{S}_{\rm eff}$ based on Lagrangian (3). For example, the effective action for the bosonic fields in the fermionic-field one-loop approximation is given by the functional integral

$$e^{i\mathcal{S}_{\rm eff}(\sigma,\pi_a,\Delta_A,\Delta^*_{A'})}=N'\int[d\bar{q}][dq]\exp\Bigl(i\int L\,d^4x\Bigr),$$

where N' is the normalization constant. Hence, it is easy to obtain

$$\mathcal{S}_{\rm eff}(\sigma,\pi_a,\Delta_A,\Delta^*_{A'})=-\int d^4x\left[\frac{\sigma^2+\pi_a^2}{4G}+\frac{\Delta_A\Delta_A^*}{4H}\right]+\widetilde{\mathcal{S}}_{\rm eff}, \tag{5}$$

where the term $\widetilde{\mathcal{S}}_{\rm eff}$ is

$$e^{i\widetilde{\mathcal{S}}_{\rm eff}}=N'\int[d\bar{q}][dq]\exp\Bigl(\frac{i}{2}\int[\bar{q}D^+q+\bar{q}^CD^-q^C-\bar{q}Kq^C-\bar{q}^CK^*q]\,d^4x\Bigr). \tag{6}$$

Here, we use the notation[2]

$$\begin{aligned}&D^+=i\gamma^\nu\partial_\nu-m_0+\mu\gamma^0-\Sigma, \qquad D^-=i\gamma^\nu\partial_\nu-m_0-\mu\gamma^0-\Sigma^{\rm t},\\ &\Sigma=\sigma+i\gamma^5\pi_a\tau_a, \qquad K^*=i\Delta^*_A\gamma^5\tau_2\lambda_A, \qquad K=i\Delta_A\gamma^5\tau_2\lambda_A, \qquad \Sigma^{\rm t}=\sigma+i\gamma^5\pi_a\tau_a^{\rm t}.\end{aligned} \tag{7}$$

For what follows, the Nambu–Gor'kov formalism in which q and q^C are combined in a new bispinor field $\Psi=\binom{q}{q^C}$ is convenient. Rewriting expression (6) in terms of Ψ and $\Psi^{\rm t}$ and subsequently integrating over bispinors in the obtained expression, we have

$$\widetilde{\mathcal{S}}_{\rm eff}(\sigma,\pi_a,\Delta_A,\Delta^*_{A'})=\frac{1}{2i}\,{\rm Tr}_{\{{\rm NG}sfcx\}}\log\begin{pmatrix}D^+ & -K\\ -K^* & D^-\end{pmatrix}\equiv\frac{1}{2i}\,{\rm Tr}_{\{{\rm NG}sfcx\}}\log Z, \tag{8}$$

where the operator Z is represented as a matrix in the two-dimensional space of Nambu–Gor'kov bispinors. Here, the trace operation ${\rm Tr}_{\{{\rm NG}sfcx\}}(\,\cdot\,)$ is taken in the Nambu–Gor'kov space as well as in the coordinate (x), color (c), spinor (s), and flavor (f) spaces.

Based on formulas (5)–(8), we can now find the system TDP $\Omega(\sigma,\pi_a,\Delta_A,\Delta^*_{A'})$,

$$\mathcal{S}_{\rm eff}\big|_{\sigma,\pi_a,\Delta_A,\Delta^*_{A'}={\rm const}}=-\Omega(\sigma,\pi_a,\Delta_A,\Delta^*_{A'})\int d^4x, \tag{9}$$

where we assume that all the bosonic fields are independent of the space coordinates x. It is well known that in thermodynamic equilibrium, i.e., in the ground state, the field means $\langle\sigma(x)\rangle\equiv\sigma^0$, $\langle\pi_a(x)\rangle\equiv\pi_a^0$, $\langle\Delta_A(x)\rangle\equiv\Delta_A^o$, and $\langle\Delta^*_{A'}(x)\rangle\equiv\Delta^{*o}_{A'}$ are the coordinates of the point of the absolute minimum of the TDP Ω. Hence, the set of x-independent parameters σ^0, π_a^0, Δ_A^o, and $\Delta^{*o}_{A'}$ provides the solution of the system of stationary equations,

$$\frac{\partial\Omega}{\partial\pi_a}=0, \qquad \frac{\partial\Omega}{\partial\sigma}=0, \qquad \frac{\partial\Omega}{\partial\Delta_A}=0, \qquad \frac{\partial\Omega}{\partial\Delta^*_{A'}}=0. \tag{10}$$

[2] To reduce the quark sector of Lagrangian (3) to the expression in the square brackets in (6), we use the well-known relations $\partial_\nu^{\rm t}=-\partial_\nu$, $C\gamma^\nu C^{-1}=-(\gamma^\nu)^{\rm t}$, $C\gamma^5C^{-1}=(\gamma^5)^{\rm t}=\gamma^5$, $\tau^2\vec{\tau}\tau^2=-(\vec{\tau})^{\rm t}$, and $\tau^2=\begin{pmatrix}0&-i\\ i&0\end{pmatrix}$.

We now make a shift in $\mathcal{S}_{\rm eff}$: $\sigma(x) \to \sigma(x) + \sigma^0$, $\pi_a(x) \to \pi_a(x) + \pi_a^0$, $\Delta_A^*(x) \to \Delta_A^*(x) + \Delta_A^{*o}$, and $\Delta_A(x) \to \Delta_A(x) + \Delta_A^o$. The matrix Z in Eq. (8) then transforms as

$$Z \to \begin{pmatrix} D_0^+ & -K_0 \\ -K_0^* & D_0^- \end{pmatrix} - \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^{\rm t} \end{pmatrix} \equiv S_0^{-1} - \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^{\rm t} \end{pmatrix}, \tag{11}$$

where S_0 is the matrix of the quark propagator in the Nambu–Gor'kov representation and

$$(K_0, K_0^*, D_0^\pm, \Sigma_0, \Sigma_0^{\rm t}) = (K, K^*, D^\pm, \Sigma, \Sigma^{\rm t})\big|_{\sigma=\sigma^0, \pi_a=\pi_a^0, \ldots}.$$

Developing Eq. (5) into a series in the bosonic fields (up to the second order), we then have

$$\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = \mathcal{S}_{\rm eff}^{(0)} + \mathcal{S}_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) + \ldots, \tag{12}$$

where (terms linear in meson and diquark fields are absent in expression (12) because of the stationarity equations)

$$\begin{aligned} \mathcal{S}_{\rm eff}^{(0)} &= -\int d^4x \left[\frac{\sigma^0\sigma^0 + \pi_a^0\pi_a^0}{4G} + \frac{\Delta_A^o\Delta_A^{*o}}{4H}\right] - \frac{i}{2}\,{\rm Tr}_{\{{\rm NG}scfx\}} \log(S_0^{-1}) \equiv \\ &\equiv -\Omega(\sigma^0, \pi_a^0, \Delta_A^o, \Delta_{A'}^{*o}) \int d^4x, \end{aligned} \tag{13}$$

$$\begin{aligned} \mathcal{S}_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = &-\int d^4x \left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A\Delta_A^*}{4H}\right] + \\ &+ \frac{i}{4}\,{\rm Tr}_{\{{\rm NG}scfx\}} \left\{ S_0 \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^{\rm t} \end{pmatrix} S_0 \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^{\rm t} \end{pmatrix} \right\}. \end{aligned} \tag{14}$$

Because we investigate the spectrum of meson and diquark excitations based on the effective action $\mathcal{S}_{\rm eff}^{(2)}$ in what follows, it is convenient to represent Eq. (14) in the form

$$\mathcal{S}_{\rm eff}^{(2)} = \mathcal{S}_{\rm mesons}^{(2)} + \mathcal{S}_{\rm diquarks}^{(2)} + \mathcal{S}_{\rm mixed}^{(2)}, \tag{15}$$

where

$$\mathcal{S}_{\rm mesons}^{(2)} = -\int d^4x\, \frac{\sigma^2 + \pi_a^2}{4G} + \frac{i}{4}\,{\rm Tr}_{scfx}\{S_{11}\Sigma S_{11}\Sigma + 2S_{12}\Sigma^{\rm t}S_{21}\Sigma + S_{22}\Sigma^{\rm t}S_{22}\Sigma^{\rm t}\}, \tag{16}$$

$$\mathcal{S}_{\rm diquarks}^{(2)} = -\int d^4x\, \frac{\Delta_A\Delta_A^*}{4H} + \frac{i}{4}\,{\rm Tr}_{scfx}\{S_{12}K^*S_{12}K^* + 2S_{11}KS_{22}K^* + S_{21}KS_{21}K\}, \tag{17}$$

$$\mathcal{S}_{\rm mixed}^{(2)} = \frac{i}{2}\,{\rm Tr}_{scfx}\{S_{11}\Sigma S_{12}K^* + S_{21}\Sigma S_{11}K + S_{12}\Sigma^{\rm t}S_{22}K^* + S_{21}KS_{22}\Sigma^{\rm t}\}, \tag{18}$$

and S_{ij} are the matrix elements of the quark propagator S_0 taken in form (11).

2.2. The TDP, the quark propagator, and the model phase structure. We recall that the phase structure of any theory is determined by its order parameter values. In our case, the order parameters are the quantities σ^0, π_a^0, Δ_A^o, and $\Delta_{A'}^{*o}$, which are the coordinates of the point of the absolute minimum of the TDP Ω and satisfy stationarity equations (10). In fact, we can substantially reduce the number of order parameters. First, we assume that the P-parity is preserved, which gives $\langle\pi_a(x)\rangle \equiv \pi_a^0 = 0$.

Second, because the TDP is invariant under color $SU_c(3)$ rotations, we can confine ourself to the case where $\langle\Delta_2(x)\rangle \equiv \Delta_2^o \neq 0$ while $\langle\Delta_{5,7}(x)\rangle \equiv \Delta_{5,7}^o = 0$. The problem of determining the phase structure of the initial model therefore reduces to finding only the two order parameters $\Delta \equiv \langle\Delta_2(x)\rangle$ and $\sigma^0 \equiv \langle\sigma(x)\rangle$. Introducing the dynamical quark mass $m \equiv m_0 + \sigma^0$, we can easily find the system TDP using relation (13) (also see [17], [18]),

$$\Omega(m,\Delta) = \frac{(m-m_0)^2}{4G} + \frac{|\Delta|^2}{4H} - 4\sum_{\pm}\int\frac{d^3q}{(2\pi)^3}|E_\Delta^\pm| - 2\sum_{\pm}\int\frac{d^3q}{(2\pi)^3}|E^\pm|, \tag{19}$$

where $E_\Delta^\pm = \sqrt{(E^\pm)^2 + |\Delta|^2}$, $E^\pm = E \pm \mu$, and $E = \sqrt{\vec{q}^{\,2} + m^2}$. Because the integrals in expression (19) contain ultraviolet divergences, we regularize them by setting $|\vec{q}| < \Lambda$, where the value of Λ is from (2). For $m_0 \neq 0$, depending on the value of μ, the absolute minimum of TDP (19) may be located either at the point $\Delta = 0$, $m \neq 0$ (in this case, the system is in the normal $SU_c(3)$-invariant phase in which CSC is absent) or at a point $\Delta \neq 0$, $m \neq 0$ corresponding to the CSC phase of the dense quark matter in which the $SU_c(3)$ symmetry is spontaneously broken to the $SU_c(2)$ symmetry. To determine which of the two phases the system is in for a given value of μ, we must know all the solutions of the stationary equations for TDP (19) (they are also called the gap equations),

$$\frac{\Delta}{4H_s} = 4i\Delta\int\frac{d^4q}{(2\pi)^4}\left\{\frac{1}{q_0^2-(E_\Delta^+)^2} + \frac{1}{q_0^2-(E_\Delta^-)^2}\right\} = 2\Delta\int\frac{d^3q}{(2\pi)^3}\left\{\frac{1}{E_\Delta^+} + \frac{1}{E_\Delta^-}\right\}, \tag{20}$$

$$\frac{m-m_0}{2G} = 4im\sum_{\pm}\int\frac{d^4q}{(2\pi)^4}\frac{1}{E}\left\{\frac{E^\pm}{q_0^2-(E^\pm)^2}\right\} + 8im\sum_{\pm}\int\frac{d^4q}{(2\pi)^4}\frac{1}{E}\left\{\frac{E^\pm}{q_0^2-(E_\Delta^\pm)^2}\right\}, \tag{21}$$

and then choose that solution for which the TDP is minimum. Of course, the described procedure can be performed in full only numerically. The simulation results are given in Fig. 1, where we depict the coordinates of the TDP absolute minimum depending on the values μ of the quark chemical potential. It is clear that for $\mu < \mu_c \approx 350\,\text{MeV}$, the system is in the normal phase because there $\Delta = 0$. For $\mu > \mu_c$, the order parameter Δ is already nonzero, which corresponds to the phase with CSC. We note the jumplike transition from the normal to the CSC phase at $\mu = \mu_c$, which is therefore a first-order phase transition point.[3]

We can use the procedure in [19] to find the matrix elements S_{ij} of the quark propagator S_0, which we need for finding effective actions (16)–(18):

$$\begin{aligned} S_{11} = &\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{q_0-E^+}{q_0^2-(E_\Delta^+)^2}\gamma^0\bar{\Lambda}_+ + \frac{q_0+E^-}{q_0^2-(E_\Delta^-)^2}\gamma^0\bar{\Lambda}_-\right\}P_{12}^{(c)} + \\ &+\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{\gamma^0\bar{\Lambda}_+}{q_0+E^+} + \frac{\gamma^0\bar{\Lambda}_-}{q_0-E^-}\right\}P_3^{(c)}, \end{aligned} \tag{22}$$

$$\begin{aligned} S_{22} = &\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{q_0-E^-}{q_0^2-(E_\Delta^-)^2}\gamma^0\bar{\Lambda}_+ + \frac{q_0+E^+}{q_0^2-(E_\Delta^+)^2}\gamma^0\bar{\Lambda}_-\right\}P_{12}^{(c)} + \\ &+\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{\gamma^0\bar{\Lambda}_+}{q_0+E^-} + \frac{\gamma^0\bar{\Lambda}_-}{q_0-E^+}\right\}P_3^{(c)}, \end{aligned} \tag{23}$$

[3] If the quark bare mass is zero, i.e., $m_0 = 0$, then for $\mu < \mu_c$, we have the phase with spontaneous breaking of the chiral $SU(2)_L \times SU(2)_R$ invariance (here the point of the TDP absolute minimum is $\Delta = 0$, $m \neq 0$). But for $\mu > \mu_c$, the absolute minimum is located at a point of form $\Delta \neq 0$, $m = 0$, i.e., the color $SU_c(3)$ symmetry spontaneously broken while the chiral $SU(2)_L \times SU(2)_R$ symmetry is restored in this domain of μ values.

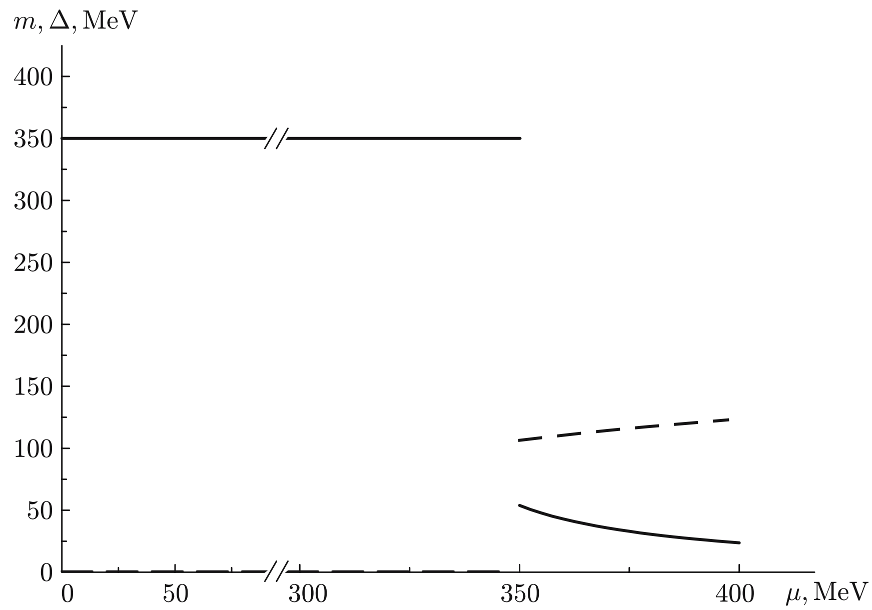

Fig. 1. The dynamical quark mass m (solid line) and the order parameter Δ (dashed line) as functions of the chemical potential μ.

$$S_{21} = -i\Delta^*\tau_2\lambda_2 \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^5\bar{\Lambda}_+}{q_0^2 - (E_\Delta^+)^2} + \frac{\gamma^5\bar{\Lambda}_-}{q_0^2 - (E_\Delta^-)^2} \right\}, \tag{24}$$

$$S_{12} = -i\Delta\tau_2\lambda_2 \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^5\bar{\Lambda}_+}{q_0^2 - (E_\Delta^-)^2} + \frac{\gamma^5\bar{\Lambda}_-}{q_0^2 - (E_\Delta^+)^2} \right\}, \tag{25}$$

where $\bar{\Lambda}_\pm = (1 \pm \gamma^0(\vec{\gamma}\vec{q} - m)/E)/2$ and $P_{12}^{(c)} = \mathrm{diag}(1,1,0)$ and $P_3^{(c)} = \mathrm{diag}(0,0,1)$ are two projection operators in the color space. We mention that both in expressions (22)–(25) and in formulas (20), (21) above, q_0 is the short notation for $q_0 + i\varepsilon\,\mathrm{sgn}(q_0)$ as $\varepsilon \to 0_+$. In the momentum representation, the poles of matrix elements (22)–(25) determine dispersion relations, i.e., the dependence of the energy q_0 of quarks propagating in a dense matter on their three-dimensional momenta. We therefore have $q_0 = E_\Delta^-$ for the energies of red/green quarks and $q_0 = E_\Delta^+$ for the energies of red/green antiquarks. Then, the energy of a blue quark or antiquark is $q_0 = E^-$ or $q_0 = E^+$. It follows from Fig. 1 that in the CSC phase where $\mu > \mu_c$ and $m < \mu$, the quantity E may reach the value μ, which is called the Fermi energy. It is obvious in this case that the creation of a red or green quark in the CSC medium requires the minimum energy $(q_0)_{\min} = |\Delta|$ on the Fermi level $E = \mu$ (in other words, we have a gap equal to $|\Delta|$ in the energy spectra of these quarks). The creation of blue quarks does not require extra energy in the CSC phase because we have $(q_0)_{\min} = 0$ at $E = \mu$ for these quarks. In the normal phase, i.e., for $\mu < \mu_c$, the creation of a quark of any color requires the minimum energy $m - \mu$. We note that in any phase, the minimum energies of antiquark and quark creation differ, which indicates that the charge symmetry is broken for $\mu \neq 0$.

3. Mesons and diquarks in a dense quark medium

We assume that Δ is a real nonnegative quantity in what follows. To find the masses of one-particle bosonic excitations over the ground state of the quark matter, we use effective action (15) determining the one-particle irreducible (1PI) two-point Green's functions of the bosonic fields $\sigma(x)$, $\pi_a(x)$, $\Delta_A(x)$, and

$\Delta^*_{A'}(x)$. We can then find particle masses from these functions. In the vacuum, i.e., when $T=0$ and $\mu=0$, the Lorentz invariance is unbroken, and all the two-particle Green's functions therefore depend on the one variable $p^2=p_0^2-\vec{p}^{\,2}$ in the momentum representation. The particle mass squared is then a zero of the corresponding 1PI Green's function (or the propagator pole) in p^2. In a dense medium (for $\mu\neq 0$), the Lorentz invariance is broken, and the two-point Green's functions therefore depend on the two variables p_0 and $\vec{p}^{\,2}$. In this case, the scalar particle mass squared is the zero of the corresponding 1PI Green's function in the variable p_0^2 in the rest frame, i.e., at $\vec{p}=0$ (see, e.g., [6], [20]). The problem of finding meson and diquark masses therefore reduces to calculating 1PI Green's functions in the momentum representation and to locating their zeros in the rest frame.

For what follows, we need a technical remark. Expression (15) implies that we have the term $\mathcal{S}^{(2)}_{\rm mixed}$ given by (18) in the effective action, which mixes the fields $\sigma(x)$, $\Delta_2(x)$, and $\Delta_2^*(x)$ and generates nondiagonal 1PI Green's functions of the form $\Gamma_{\sigma\phi}(p)$ with one external leg corresponding to the field σ and the other corresponding to the field ϕ ($\phi=\Delta_2^*,\Delta_2$). But detailed calculations demonstrate that in the rest frame, i.e., at $p=(p_0,0,0,0)$, each such function is proportional to $m\Delta$ (see [7]). Hence, in the normal quark phase, where $\Delta=0$ and the color symmetry is unbroken, there is no mixing of σ-mesons with diquarks. At the same time, when calculating particle masses in the CSC phase, we neglect nondiagonal Green's functions of the form $\Gamma_{\sigma\phi}(p)$ because the dynamical quark masses m are relatively small compared with Δ (see Fig. 1).[4]

3.1. Masses of π- and σ-mesons. The 1PI Green's functions for σ- and π-mesons can be found from expression (16) by formulas

$$\Gamma(x-y)=-\frac{\delta^2\mathcal{S}^{(2)}_{\rm mesons}}{\delta\sigma(y)\delta\sigma(x)},\qquad \Pi_{ab}(x-y)=-\frac{\delta^2\mathcal{S}^{(2)}_{\rm mesons}}{\delta\pi_b(y)\delta\pi_a(x)}. \tag{26}$$

Substituting formulas (22)–(25) for the matrix elements S_{ij} in expression (16) and using relations (26), we can easily obtain the 1PI Green's functions $\Gamma(x-y)$ and $\Pi_{ab}(x-y)$ and then the corresponding expressions $\Gamma(p)$ and $\Pi_{ab}(p)$ in the momentum representation. The zeros of $\Gamma(p)$ and $\Pi_{ab}(p)$ determine the dispersion relations for particles and antiparticles. Because we are interested only in particle masses in the dense quark medium, we must pass to the rest frame in which the zeros of the functions $\Gamma(p_0)$ and $\Pi_{ab}(p_0)$ are the masses of the corresponding particles, as stated above. Therefore, if $\vec{p}=0$, then

$$\begin{aligned}\Pi_{ab}(p_0)&=\frac{\delta_{ab}}{2G}-8\delta_{ab}\int\frac{d^3q}{(2\pi)^3}\frac{E^+_\Delta E^-_\Delta+E^+E^-+\Delta^2}{E^+_\Delta E^-_\Delta}\frac{E^+_\Delta+E^-_\Delta}{(E^+_\Delta+E^-_\Delta)^2-p_0^2}-\\&\quad-16\delta_{ab}\int\frac{d^3q}{(2\pi)^3}\frac{\theta(E-\mu)E}{4E^2-p_0^2}\equiv\delta_{ab}\Pi(p_0),\end{aligned}\tag{27}$$

$$\Gamma(p_0)=\Gamma_0(p_0^2)+\Gamma_1(p_0^2),\tag{28}$$

$$\begin{aligned}\Gamma_0(p_0^2)&=\frac{1}{2G}-8\int\frac{d^3q}{(2\pi)^3}\frac{\vec{q}^{\,2}}{E^2}\frac{E^+_\Delta E^-_\Delta+E^+E^-+\Delta^2}{E^+_\Delta E^-_\Delta}\frac{E^+_\Delta+E^-_\Delta}{(E^+_\Delta+E^-_\Delta)^2-p_0^2}-\\&\quad-16\int\frac{d^3q}{(2\pi)^3}\frac{\vec{q}^{\,2}}{E}\frac{\theta(E-\mu)}{4E^2-p_0^2},\end{aligned}\tag{29}$$

$$\Gamma_1(p_0^2)=16\Delta^2m^2\int\frac{d^3q}{(2\pi)^3E^2}\left\{\frac{1}{E^+_\Delta[p_0^2-4(E^+_\Delta)^2]}+\frac{1}{E^-_\Delta[p_0^2-4(E^-_\Delta)^2]}\right\},\tag{30}$$

[4]Moreover, if the bare quark mass m_0 vanishes, then $m=0$ in the CSC phase as well. The mixing between σ-mesons and diquarks therefore does not occur at all in the CSC phase in the chiral limit.

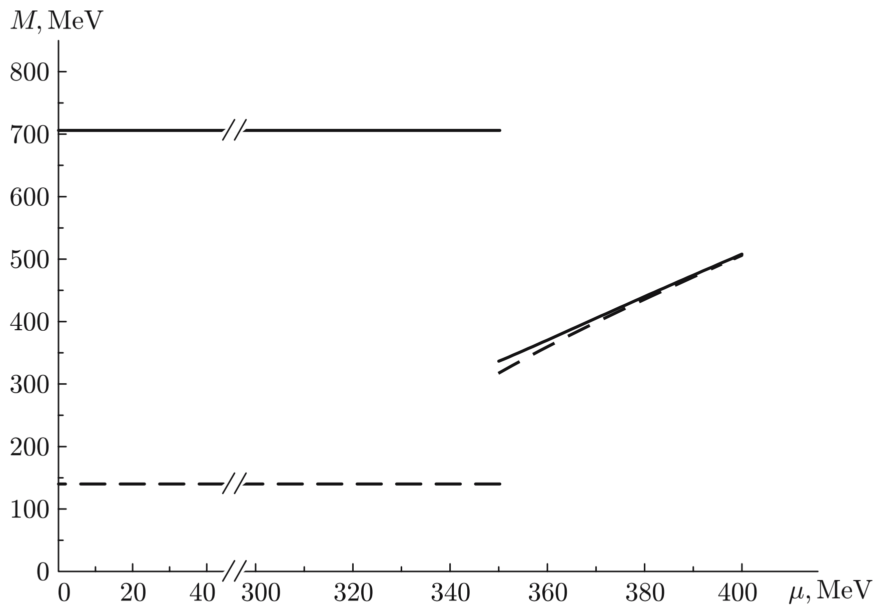

Fig. 2. The masses of the σ-meson (solid line) and π-meson (dashed line) as functions of μ.

where we use the same notation as in expression (19). The values p_0 at which functions (27) and (28) vanish are the respective masses of π- and σ-mesons (see Fig. 2).[5]

It follows from Fig. 2 that the masses of π- and σ-mesons are of the order of 300 MeV in the CSC phase. Moreover, we stress that the π-meson is stable w.r.t. the strong decay channels in this phase. If we neglect the mixing between the σ-meson and the Δ_2-diquark (this is the case we consider here), then the σ-meson is also a stable particle in the CSC phase. But we note that if we take the mixing in the CSC phase into account, then the σ-meson is a resonance with a relatively small width of the order of 30 MeV (see [7]).

3.2. Diquark masses. For diquark masses, we must begin with effective action (17), which can be represented in the form $\mathcal{S}^{(2)}_{\text{diquarks}} = \mathcal{S}^{(2)}_{22} + \mathcal{S}^{(2)}_{55} + \mathcal{S}^{(2)}_{77}$, where

$$\mathcal{S}^{(2)}_{22} = -\int d^4x \, \frac{\Delta_2\Delta_2^*}{4H} + \frac{i}{4}\,\mathrm{Tr}_{scfx}\{S_{12} i\Delta_2^*\gamma^5\tau_2\lambda_2 S_{12} i\Delta_2^*\gamma^5\tau_2\lambda_2 + \\ + 2S_{11} i\Delta_2\gamma^5\tau_2\lambda_2 S_{22} i\Delta_2^*\gamma^5\tau_2\lambda_2 + S_{21} i\Delta_2\gamma^5\tau_2\lambda_2 S_{21} i\Delta_2\gamma^5\tau_2\lambda_2\}, \tag{31}$$

$$\mathcal{S}^{(2)}_{AA} = -\int d^4x \, \frac{\Delta_A\Delta_A^*}{4H} + \frac{i}{2}\,\mathrm{Tr}_{scfx}\{S_{11} i\Delta_A\gamma^5\tau_2\lambda_A S_{22} i\Delta_A^*\gamma^5\tau_2\lambda_A\}, \tag{32}$$

$A = 5, 7$. Hence, diquarks do not mix with each other and each of expressions (31) and (32) can be represented in the form ($A = 2, 5, 7$)

$$\mathcal{S}^{(2)}_{AA} = -\frac{1}{2}\sum_{X,Y}\int d^4u\, d^4v\, X(u)\, {}_A\Gamma_{XY}(u-v) Y(v), \tag{33}$$

where for each fixed $A = 2, 5, 7$, $X(x), Y(x) = \Delta_A(x), \Delta_A^*(x)$, and ${}_A\Gamma_{XY}(z)$ are the matrix elements of the $2{\times}2$ matrix ${}_A\Gamma(z)$ of the 1PI Green's functions (we use the coordinate representation in expression (33))

[5]We note that the term $\Gamma_1(p_0^2)$ in formula (28) is proportional to $\Delta^2 m^2$ and is therefore a quantity of a higher order of smallness than the nondiagonal 1PI Green's functions $\Gamma_{\sigma\phi}(p_0)$ ($\phi = \Delta_2^*, \Delta_2$). Because we neglect the latter by convention, we also omit term (30) in the numerical calculations of the σ-meson mass.

of the diquark fields $\Delta_A(x)$ and $\Delta_A^*(x)$. This matrix is nondiagonal in general, and the diquark masses are therefore zeros of its determinant in the momentum representation.

3.2.1. Diquark masses in the CSC phase ($\boldsymbol{\Delta \neq 0,\ \mu > \mu_c}$). We first consider the diquark sector Δ_5 and Δ_5^*. By virtue of (33) at $A = 5$, we have

$$ {}_5\Gamma_{XY}(x-y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{55}}{\delta Y(y)\delta X(x)} \tag{34} $$

(here $X, Y = \Delta_5(x), \Delta_5^*(x)$). We recall that ${}_5\Gamma(z)$ is a symmetric matrix, i.e., ${}_5\Gamma_{XY}(z) = {}_5\Gamma_{YX}(-z)$. Expressions (32) and (34) imply that ${}_5\Gamma_{\Delta_5\Delta_5}(z) = {}_5\Gamma_{\Delta_5^*\Delta_5^*}(z) = 0$, and the nonzero matrix elements in the momentum representation are

$$ {}_5\Gamma_{\Delta_5^*\Delta_5}(p) = \frac{1}{4H} - i\,\mathrm{Tr}_{sc}\int \frac{d^4q}{(2\pi)^4}\{S_{11}(q+p)i\gamma^5\lambda_5 S_{22}(q)i\gamma^5\lambda_5\}, \tag{35} $$

${}_5\Gamma_{\Delta_5\Delta_5^*}(p) = {}_5\Gamma_{\Delta_5^*\Delta_5}(-p)$, where $S_{11}(q)$ and $S_{22}(q)$ are the Fourier transforms of expressions (22) and (23). Because we are interested in the diquark masses, we must pass to the rest frame $p = (p_0, 0, 0, 0)$ in expression (35) (see [6], [20]). It is then easy to obtain

$$ {}_5\Gamma_{\Delta_5^*\Delta_5}(p_0) = \frac{1}{4H} - 4i\int \frac{d^4q}{(2\pi)^4}\left\{\frac{q_0 + E^+}{(p_0+q_0+E^+)(q_0^2-(E_\Delta^+)^2)} + \frac{q_0 - E^-}{(p_0+q_0-E^-)(q_0^2-(E_\Delta^-)^2)}\right\}. \tag{36} $$

This expression holds for both $\Delta = 0$ and $\Delta \neq 0$. For $\Delta \neq 0$, i.e., in the CSC phase, it is useful to exclude the coupling constant H from expression (36) using gap equation (20), subsequently integrating over q_0 using the prescription $q_0 \to q_0 + i\varepsilon\,\mathrm{sgn}(q_0)$, $(p_0+q_0) \to (p_0+q_0) + i\varepsilon\,\mathrm{sgn}(p_0+q_0)$. As a result, we obtain

$$ {}_5\Gamma_{\Delta_5^*\Delta_5}(p_0) = 2p_0 H(p_0), \qquad {}_5\Gamma_{\Delta_5\Delta_5^*}(p_0) = -2p_0 H(-p_0), \tag{37} $$

where

$$ H(p_0) = \int \frac{d^3q}{(2\pi)^3}\left\{\frac{1}{(p_0+E^++E_\Delta^+)E_\Delta^+} + \frac{\theta(E^-)}{(p_0-E^--E_\Delta^-)E_\Delta^-} + \frac{\theta(-E^-)}{(p_0-E^-+E_\Delta^-)E_\Delta^-}\right\} \tag{38} $$

(we take $\mu > 0$ into account in deriving this equation). Using formulas (37), we can now easily obtain the determinant of the matrix ${}_5\Gamma(p_0)$:

$$ \det {}_5\Gamma(p_0) = -{}_5\Gamma_{\Delta_5^*\Delta_5}(p_0)\,{}_5\Gamma_{\Delta_5\Delta_5^*}(p_0) \equiv 4p_0^2 H(p_0)H(-p_0). \tag{39} $$

We recall that the value of p_0^2 where $\det {}_5\Gamma(p_0)$ vanishes is the mass squared of the bosonic excitation in the Δ_5 sector of the theory. Because $\Delta_5(x)$ is a complex-valued field (and has two real degrees of freedom), the equation $\det {}_5\Gamma(p_0) = 0$ must have either two different solutions in the variable p_0^2 (corresponding to two particles with different masses) or one doubly degenerate solution (corresponding to two distinct particles

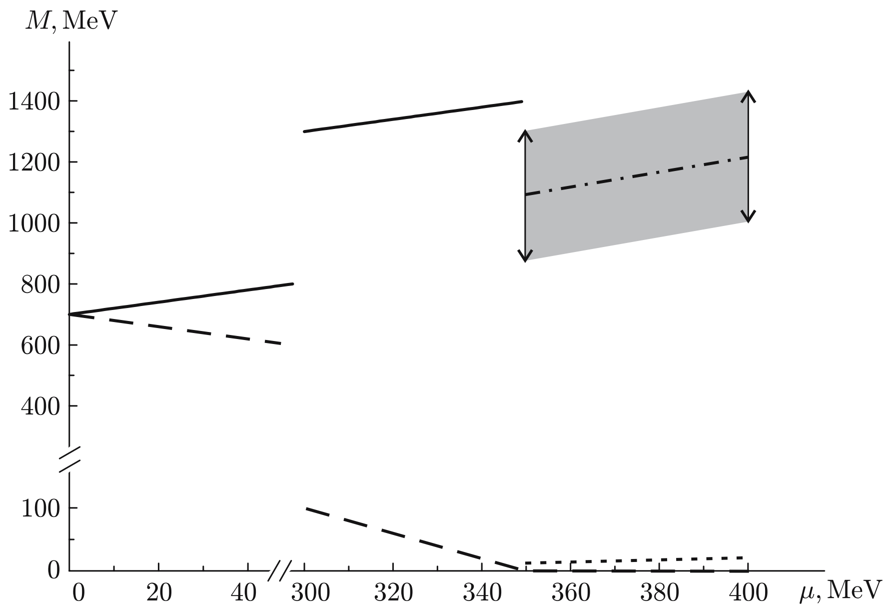

Fig. 3. Diquark masses: For $\mu < \mu_c = 350\,\mathrm{MeV}$, six diquark states constitute the $SU_c(3)$ triplet and antitriplet. Heavy particles with the mass M_{Δ^*} (solid line) enter the triplet, and light particles with the mass M_Δ (dashed line) enter the antitriplet (see (46)). In the CSC phase (for $\mu > \mu_c$), the mass spectrum in the diquark sector contains three GBs (dashed line), the $SU_c(2)$ doublet of light diquarks (dotted line), and the color-singlet heavy resonance. Its mass M is represented by the dot-dashed line. The resonance width Γ is imitated in the figure by the shadowed parallelogram of width Γ; its upper and lower boundaries are $\Gamma/2$ higher and lower than the resonance mass M.

with the same mass). This equation obviously admits one trivial solution $p_0^2 = 0$, which can be identified with the massless Goldstone boson (GB). (Because $H(0) \neq 0$ for $\mu \neq 0$, no other massless bosons arise in the Δ_5 sector.) In contrast to Lorentz-invariant systems, where massless particles obey a linear dispersion law, this boson obeys the quadratic dispersion law, i.e., $p_0 \sim \vec{p}^{\,2}$ as $|\vec{p}| \to 0$ [5]. Further, we assume that the function $H(p_0)$ becomes zero at a nonzero point $p_0 = -m_1$, i.e., $H(-m_1) = 0$. The determinant $\det {}_5\Gamma(p_0)$ then vanishes at two points $p_0 = \pm m_1$ in the variable p_0 (and vanishes at the point $p_0^2 = m_1^2$ in the variable p_0^2), which corresponds to a boson of mass m_1. Numerical calculations demonstrate that m_1 is a very small quantity in the CSC phase: $m_1 \sim 15\,\mathrm{MeV}$.

We have exactly the same picture in the Δ_7 sector of the theory, where we have a massless GB term and a light boson of mass m_1. Because the CSC-phase ground state is $SU_c(2)$ symmetric, we can conclude that we have found the $SU_c(2)$ doublet of GBs and the $SU_c(2)$ doublet of light bosons with the mass m_1 in the Δ_5,Δ_7 sector of the theory. The light boson doublet mass m_1 is depicted by the dotted line in Fig. 3.

We now investigate the spectrum of bosonic excitations of the CSC phase in the diquark Δ_2,Δ_2^* sector. For these fields, the matrix of 1PI Green's functions ${}_2\Gamma(p_0)$ has the following structure in the rest frame in the momentum representation:

$$\begin{aligned} {}_2\Gamma_{\Delta_2\Delta_2}(p_0) &= {}_2\Gamma_{\Delta_2^*\Delta_2^*}(p_0) = 4\Delta^2 I_0(p_0^2), \\ {}_2\Gamma_{\Delta_2\Delta_2^*}(p_0) &= {}_2\Gamma_{\Delta_2^*\Delta_2}(-p_0) = (4\Delta^2 - 2p_0^2)I_0(p_0^2) + 4p_0 I_1(p_0^2), \end{aligned} \tag{40}$$

where

$$I_0(p_0^2) = \int \frac{d^3q}{(2\pi)^3}\frac{1}{E_\Delta^+[4(E_\Delta^+)^2 - p_0^2]} + \int \frac{d^3q}{(2\pi)^3}\frac{1}{E_\Delta^-[4(E_\Delta^-)^2 - p_0^2]}, \tag{41}$$

$$I_1(p_0^2) = \int \frac{d^3q}{(2\pi)^3} \frac{E^+}{E_\Delta^+[4(E_\Delta^+)^2 - p_0^2]} - \int \frac{d^3q}{(2\pi)^3} \frac{E^-}{E_\Delta^-[4(E_\Delta^-)^2 - p_0^2]}. \tag{42}$$

The mass spectrum is determined by the equation

$$\det {}_2\Gamma(p_0) \equiv 4p_0^2\{(p_0^2 - 4\Delta^2)I_0^2(p_0^2) - 4I_1^2(p_0^2)\} = 0. \tag{43}$$

This equation admits an obvious solution $p_0^2 = 0$ in the variable p_0^2, which corresponds to the GB. Detailed studies (see [6]) demonstrated that the second solution of Eq. (43) corresponding to a heavy resonance lies on the second sheet of the Riemann surface of the variable p_0^2. Its mass M and width Γ are depicted in Fig. 3 for $\mu > \mu_c$. In the diquark Δ_2,Δ_2^* sector, both the GB and the heavy resonance are singlets w.r.t. $SU_c(2)$ group.

We mention an interesting feature of the CSC phase in which we have a nonstandard number of GBs. It is well known that if an internal symmetry group $\mathcal{G}$ of dimension r_1 is spontaneously broken to a subgroup $\mathcal{H}$ of dimension r_2 in a Lorentz-invariant theory, then $r_1 - r_2$ GBs must be present in the mass spectrum (the Goldstone theorem). Because $\mathcal{G} \equiv SU_c(3)$ and $\mathcal{H} \equiv SU_c(2)$ in our case, we have $r_1 - r_2 = 5$. But we find only three GBs, which contradicts the Goldstone theorem. The resolution of this paradox is as follows. The Goldstone theorem is formulated for Lorentz-invariant systems, while the term with the chemical potential in Lagrangian (1) explicitly breaks the Lorentz symmetry. In such a situation, the Nielsen–Chadha theorem [21] should be applied. Let n_1 and n_2 be the numbers of massless excitations with the respective linear ($E \sim |\vec{p}|$) and quadratic ($E \sim |\vec{p}|^2$) dispersion laws in the domain of small three-dimensional momentum $\vec{p}$ (E is the particle energy). Then $r_1 - r_2 \leq n_1 + 2n_2$. Recalling that two of the three massless excitations obey the quadratic dispersion law in the CSC phase of model (1), we see that our results do not contradict this more general theorem. A nonstandard number of GBs is a feature of other quantum theories with a chemical potential [22].

All that was said above also holds for quark matter without color neutrality. To describe a color-neutral CSC medium, we must introduce a term with the color chemical potential in Lagrangian (1) (see, e.g., [7], [23]). Taking the new term into account, we reduce the color symmetry of the model to the group $\mathcal{G} \equiv SU_c(2) \times U_{\lambda_8}(1)$, for which $r_1 = 4$. In the CSC phase of this system, the symmetry of the ground state is still $\mathcal{H} \equiv SU_c(2)$, i.e., $r_2 = 3$. Hence, $r_1 - r_2 = 1$. This (standard) number of GBs equal to one was found in the mass spectrum of the color-neutral CSC phase of quark matter in [7], [23]. Moreover, we have four (instead of two) light diquark states and a heavy resonance in the diguark sector of the model, while the meson masses remain practically unchanged if we impose the color-neutrality condition [7], [23].

3.2.2. Diquark masses in the normal phase ($\Delta = 0$, $\mu < \mu_c$). In the $SU_c(3)$-symmetric phase ($\mu < \mu_c$), the order parameter Δ is zero, and the three diquark fields $\Delta_A(x)$ ($A = 2, 5, 7$) mix neither with each other nor with meson fields in effective action (15). It therefore suffices to consider the bosonic excitation spectrum, for example, in the Δ_5 sector. In this phase, the determinant of the ${}_5\Gamma(p_0)$ matrix of the 1PI Green's functions of diquark fields Δ_5, Δ_5^* is (in the momentum representation in the rest frame)

$$\det {}_5\Gamma(p_0) = {}_5\Gamma_{\Delta_5^*\Delta_5}(p_0)\, {}_5\Gamma_{\Delta_5\Delta_5^*}(p_0) = {}_5\Gamma_{\Delta_5^*\Delta_5}(p_0)\, {}_5\Gamma_{\Delta_5^*\Delta_5}(-p_0), \tag{44}$$

where ${}_5\Gamma_{\Delta_5^*\Delta_5}(p_0)$ is given by expression (36). (Equality (39) for $\det {}_5\Gamma(p_0)$ holds only for $\Delta \neq 0$, i.e., in the CSC phase; expression (39) is inapplicable in the $SU_c(3)$-symmetric phase, where $\Delta = 0$.) By virtue of the relation $m > \mu$, which holds for $\mu < \mu_c$ (see Fig. 1), we see that $E \equiv \sqrt{\vec{q}^{\,2} + m^2} > \mu$, i.e., $E^\pm > 0$, in the normal phase. In this case, we can easily integrate over q_0 in expression (36), thus obtaining an expression valid only in the $SU_c(3)$-symmetric phase:

$${}_5\Gamma_{\Delta_5^*\Delta_5}(p_0) = \frac{1}{4H} - 16 \int \frac{d^3q}{(2\pi)^3} \frac{E}{4E^2 - (p_0 + 2\mu)^2} \equiv \frac{1}{4H} - F(\epsilon), \tag{45}$$

where $\epsilon = (p_0 + 2\mu)^2$. Diquark masses are obviously determined by the equation $\det {}_5\Gamma(p_0) = 0$, i.e., by zeros of expression (45) in which the function $F(\epsilon)$ is analytic in the complex ϵ plane everywhere except the cut along the real axis $4m^2 < \epsilon$. Simulations show that for the model parameter values in (2), the equation ${}_5\Gamma_{\Delta_5^*\Delta_5}(p_0) = 0$ has only the root ϵ_0 on the real axis ($0 < \epsilon_0 < 4m^2$), which results in the diquark mass values (solutions of Eq. (44)) for $\mu < \mu_c$

$$(M_\Delta)^2 = (1.998m - 2\mu)^2, \qquad (M_{\Delta^*})^2 = (1.998m + 2\mu)^2. \tag{46}$$

We identify M_Δ with the mass of a diquark with the baryon charge $B = 2/3$ and M_{Δ^*} with the mass of an antidiquark with the baryon charge $B = -2/3$. The diquark and antidiquark masses differ because the charge symmetry is absent for $\mu \neq 0$. Because of the $SU_c(3)$ symmetry, particles with masses (46) are also present in the sectors Δ_2 and Δ_7. In the model spectrum for $\mu < \mu_c$, we therefore have the $SU(3)_c$ antitriplet of diquarks of mass M_Δ and the $SU(3)_c$ triplet of antidiquarks of mass M_{Δ^*}. The dependence of these masses on μ is shown in Fig. 3 for $\mu < \mu_c$.

We recall that we have considered only the case $H = 3G/4$ up to now. But it is interesting to investigate how the quantity H (for a fixed G) affects the diquark mass spectrum in the normal phase. Formula (45) implies that the zero ϵ_0 of this expression falls in the interval $0 < \epsilon_0 < 4m^2$ only if $H^* < H < H^{**}$, where

$$H^* \equiv \frac{1}{4F(4m^2)} = \frac{\pi^2}{4[\Lambda\sqrt{m^2+\Lambda^2} + m^2\log((\Lambda + \sqrt{m^2+\Lambda^2})/m)]},$$

$$H^{**} \equiv \frac{1}{4F(0)} = \frac{\pi^2}{4[\Lambda\sqrt{m^2+\Lambda^2} - m^2\log((\Lambda + \sqrt{m^2+\Lambda^2})/m)]} = \frac{3mG}{2(m-m_0)}. \tag{47}$$

In this case, stable diquarks whose masses are qualitatively described by formulas (46) are present in the normal phase. If the interaction in the diquark channel is relatively small, i.e., if $H < H^*$, then the zero ϵ_0 drifts to the second Riemann sheet of the variable ϵ, and we consequently obtain nonstable diquark modes (resonances). In the other limiting case, i.e., with strong interaction in the diquark channel ($H > H^{**}$), the zero ϵ_0 of expression (45) is located on the negative half-axis of the variable ϵ, i.e., $(p_0+2\mu)^2 < 0$. This indicates a tachyonic instability of the normal phase and is sufficient evidence for the existence of another, lower ground state of a system. Indeed, as was shown in [24], the color symmetry of the model is broken for large values of the coupling constant H even at $\mu = 0$; hence, the CSC phase then prevails over the normal quark phase for all $\mu > 0$.

4. Conclusion

Our main aim in this paper was to describe dense quark matter in the framework of NJL model (1) containing both quark–antiquark and diquark interaction channels. Investigating the TDP, we showed that for model parameter values (2), the CSC phase is realized in the model for $\mu > \mu_c = 350\,\text{MeV}$. Because the diquark condensate is nonzero in the ground state of this phase, the initial $SU_c(3)$ symmetry is spontaneously broken to $SU_c(2)$. In this case, if the Goldstone theorem is used to find the number of GBs in the mass spectrum, then five GBs would be expected. But we showed that we have a nonstandard number in the CSC phase, i.e., three GBs, two of them obeying a quadratic dispersion law. This is not contradictory, because in Lorentz-noninvariant systems such as NJL model (1), massless bosons with a quadratic dependence of the energy on the momentum must be counted twice when calculating the number of GBs [21]. In addition to three GBs in the diquark sector of the model CSC phase, we found a doublet of light states (with a mass $\sim 15\,\text{MeV}$) and a heavy resonance with a mass $\sim 1100\,\text{MeV}$.

Concerning mesonic excitations in the CSC phase, we proved that stable π-mesons with a mass $\sim 330\,\text{MeV}$ are present. The σ-meson has approximately the same mass but is stable only in the chiral limit in which the current quark mass $m_0 = 0$. If $m_0 \neq 0$, then, first, the σ-meson mixes with diquarks in the CSC phase; second, it is a nonstable particle with a decay width $\sim 30\,\mathrm{MeV}$.

We also considered diquark masses in the normal phase in which the $SU_c(3)$ symmetry is unbroken ($\mu < \mu_c$). In this case, for model parameter values (2), all the diquarks are stable particles, among which three (antidiquarks) constitute a heavy $SU_c(3)$ triplet and the other three (diquarks) constitute a light antitriplet (see formulas (46) and Fig. 3). The diquark and antidiquark masses differ because the charge symmetry is absent for $\mu \neq 0$. If the coupling constant H is sufficiently small, then all diquarks become nonstable states in the normal phase. If the interaction in the diquark channel is strong ($H > H^{**}$, where the quantity H^{**} is given by formula (47)), then the normal quark phase is prohibited in the initial NJL model, and the color symmetry is necessarily spontaneously broken.

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REFERENCES

1. B. C. Barrois, *Nucl. Phys. B*, **129**, 390 (1977); D. Bailin and A. Love, *Nucl. Phys. B*, **190**, 175 (1981).
2. D. T. Son, *Phys. Rev. D*, **59**, 094019 (1999); T. Schäfer and F. Wilczek, *Phys. Rev. D*, **60**, 114033 (1999); D. K. Hong et al., *Phys. Rev. D*, **61**, 056001 (2000); W. E. Brown, J. T. Liu, and H.-C. Ren, *Phys. Rev. D*, **61**, 114012 (2000).
3. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.*, **124**, 246 (1961).
4. R. Rapp et al., *Ann. Phys.*, **280**, 35 (2000); M. Alford, *Annual Rev. Nucl. Part. Sci.*, **51**, 131 (2001); G. Nardulli, *Riv. Nuovo Cimento*, **25**, 1 (2002); M. Buballa, *Phys. Rep.*, **407**, 205 (2005); M. Huang, *Internat. J. Mod. Phys. E*, **14**, 675 (2005); I. A. Shovkovy, *Found. Phys.*, **35**, 1309 (2005).
5. D. Blaschke et al., *Phys. Rev. D*, **70**, 014006 (2004).
6. D. Ebert, K. G. Klimenko, and V. L. Yudichev, *Phys. Rev. C*, **72**, 015201 (2005).
7. D. Ebert, K. G. Klimenko, and V. L. Yudichev, *Phys. Rev. D*, **72**, 056007 (2005).
8. N. N. Bogoliubov, A. A. Logunov, A. I. Oksak, and I. T. Todorov, *General Principles of Quantum Field Theory* [in Russian], Nauka, Moscow (1987); English transl., Kluwer, Dordrecht (1990).
9. M. K. Volkov and D. Ebert, *Sov. J. Nucl. Phys.*, **36**, 736 (1982); *Z. Phys. C*, **16**, 205 (1983); M. K. Volkov, *Ann. Phys.*, **157**, 282 (1984); D. Ebert and H. Reinhardt, *Nucl. Phys. B*, **271**, 188 (1986); R. G. Jafarov and V. E. Rochev, *Central Eur. J. Phys.*, **2**, 367 (2004); R. G. Jafarov, *Azerbaijan J. Phys.*, **2005**, No. 3, 27 (2005); "On pion correction to quark mass in next-leading order of mean-field expansion in NJL model," hep-ph/0412114 (2004).
10. M. K. Volkov, *Sov. J. Part. Nucl.*, **17**, 186 (1986).
11. T. Hatsuda and T. Kunihiro, *Phys. Rep.*, **247**, 221 (1994); S. P. Klevansky, *Rev. Modern Phys.*, **64**, 649 (1992).
12. D. Ebert, L. Kaschluhn, and G. Kastelewicz, *Phys. Lett. B*, **264**, 420 (1991); U. Vogl, *Z. Phys. A*, **337**, 191 (1990); U. Vogl and W. Weise, *Progr. Part. Nucl. Phys.*, **27**, 195 (1991).
13. D. Ebert and L. Kaschluhn, *Phys. Lett. B*, **297**, 367 (1992); D. Ebert and T. Jurke, *Phys. Rev. D*, **58**, 034001 (1998); L. J. Abu-Raddad et al., *Phys. Rev. C*, **66**, 025206 (2002); H. Reinhardt, *Phys. Lett. B*, **244**, 316 (1990); N. Ishii, W. Benz, and K. Yazaki, *Phys. Lett. B*, **301**, 165 (1993); C. Hanhart and S. Krewald, *Phys. Lett. B*, **344**, 55 (1995).
14. V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, *Phys. Lett. B*, **349**, 477 (1995); A. S. Vshivtsev, K. G. Klimenko, and B. V. Magnitskii, *JETP Letters*, **62**, 283 (1995); M. A. Vdovichenko, A. S. Vshivtsev, and K. G. Klimenko, *Phys. Atomic Nuclei*, **63**, 542 (2000); D. Ebert et al., *Phys. Rev. D*, **61**, 025005 (2000); T. Inagaki, D. Kimura, and T. Murata, *Progr. Theor. Phys.*, **111**, 371 (2004); E. J. Ferrer, V. de la Incera, and C. Manuel, *Phys. Rev. Lett.*, **95**, 152002 (2005).

15. T. Inagaki, T. Muta, and S. D. Odintsov, *Progr. Theoret. Phys.* (Suppl.), **127**, 93 (1997); E. V. Gorbar, *Phys. Rev. D*, **61**, 024013 (2000); Yu. I. Shil'nov and V. V. Chitov, *Phys. Atomic Nuclei*, **64**, 2051 (2001); X. Huang, X. Hao, and P. Zhuang, "Chiral phase structure at finite temperature and density in Einstein universe," hep-ph/0602186 (2006); D. K. Kim and I. G. Koh, *Phys. Rev. D*, **51**, 4573 (1995); A. S. Vshivtsev, M. A. Vdovichenko, and K. G. Klimenko, *JETP*, **87**, 229 (1998); A. S. Vshivtsev, A. K. Klimenko, and K. G. Klimenko, *Phys. Atomic Nuclei*, **61**, 479 (1998); E. J. Ferrer, V. de la Incera, and V. P. Gusynin, *Phys. Lett. B*, **455**, 217 (1999); E. J. Ferrer and V. de la Incera, *Vestn. Tomskogo Gos. Ped. Univ.*, **44**, No. 7, 88 (2004); hep-ph/0408229 (2004).
16. M. Asakawa and K. Yazaki, *Nucl. Phys. A*, **504**, 668 (1989); P. Zhuang, J. Hüfner, and S. P. Klevansky, *Nucl. Phys. A*, **576**, 525 (1994); D. Ebert et al., *Internat. J. Mod. Phys. A*, **8**, 1295 (1993); M. Buballa, *Nucl. Phys. A*, **611**, 393 (1996); A. S. Vshivtsev and K. G. Klimenko, *JETP Letters*, **64**, 338 (1996); D. Ebert and K. G. Klimenko, *Nucl. Phys. A*, **728**, 203 (2003).
17. T. M. Schwarz, S. P. Klevansky, and G. Papp, *Phys. Rev. C*, **60**, 055205 (1999); M. Sadzikowski, *Modern Phys. Lett. A*, **16**, 1129 (2001); D. Ebert, K. G. Klimenko, and H. Toki, *Phys. Rev. D*, **64**, 014038 (2001); D. Ebert, V. V. Khudyakov, V. Ch. Zhukovsky, and K. G. Klimenko, *Phys. Rev. D*, **65**, 054024 (2002); T. Ohsaku, *Phys. Lett. B*, **634**, 285 (2006); hep-ph/0601009 (2006).
18. M. Huang, P. Zhuang, and W. Chao, *Phys. Rev. D*, **67**, 065015 (2003).
19. M. Huang, P. Zhuang, and W. Chao, *Phys. Rev. D*, **65**, 076012 (2002); Z. G. Wang, S. L. Wan, and W. M. Yang, "New approach for calculating Nambu–Gorkov propagator in color superconductivity theory," hep-ph/0508302 (2005).
20. J. B. Kogut et al., *Nucl. Phys. B*, **582**, 477 (2000); P. Costa et al., *Phys. Rev. C*, **70**, 025204 (2004).
21. H. B. Nielsen and S. Chadha, *Nucl. Phys. B*, **105**, 445 (1976).
22. V. A. Miransky and I. A. Shovkovy, *Phys. Rev. Lett.*, **88**, 111601 (2002); V. P. Gusynin, V. A. Miransky, and I. Shovkovy, *Phys. Lett. B*, **581**, 82 (2004); *Modern Phys. Lett. A*, **19**, 1341 (2004); T. Brauner, *Phys. Rev. D*, **72**, 076002 (2005); L. He, M. Jin, and P. Zhuang, *Phys. Rev. A*, **74**, 033604 (2006); cond-mat/0604580 (2006).
23. L. He, M. Jin, and P. Zhuang, "Neutral color superconductivity and pseudo-Goldstone modes," hep-ph/0504148 (2005).
24. V. Ch. Zhukovsky, K. G. Klimenko, V. V. Khudyakov, and D. Ebert, *JETP Letters*, **74**, 523 (2001); D. Ebert, V. V. Khudyakov, K. G. Klimenko, H. Toki, and V. Ch. Zhukovsky, "Chromomagnetic catalysis of chiral symmetry breaking and color superconductivity," hep-ph/0108185 (2001).