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Diquarks in the color-flavor locked phase of dense quark matter

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Diquark excitations of dense quark matter are considered in the framework of the Nambu–Jona-Lasinio model with three types of massless quarks in the presence of a quark number chemical potential μ. We investigate the effective action of meson and diquark fields at sufficiently high values of μ, where the color-flavor locked (CFL) phase is realized, and prove the existence of NG bosons in the sector of pseudoscalar diquarks. In the sector of scalar diquarks an additional NG boson is found, corresponding to the spontaneous breaking of the $\mathrm{U}(1)_B$ baryon symmetry in the CFL phase. Finally, the existence of massive scalar and pseudoscalar diquark excitations is demonstrated.

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I. INTRODUCTION

It is well-known that at sufficiently high baryon densities massless three-flavor QCD is in the so-called color-flavor locked (CFL) phase [1,2]. In this phase the original $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_c \times \mathrm{U}(1)_B$ symmetry of QCD is spontaneously broken down to the diagonal $\mathrm{SU}(3)_{L+R+c}$ subgroup. Correspondingly, seventeen massless excitations must appear in the mass spectrum of the theory. Eight of them might be used to ensure a mass of gluons by the Anderson–Higgs mechanism. The properties of the remaining nine, one scalar and eight pseudoscalar Nambu–Goldstone (NG) bosons, as well as other collective modes of the CFL phase were studied already in the framework of weak-coupling QCD [3–5].

It is clear that a weak-coupling QCD analysis of the color superconductivity phenomena, including the CFL one, can only be trusted at rather high baryon densities or, correspondingly, for values of the quark number chemical potential $\mu \gg 1$ GeV [6]. At moderate values of $\mu \sim 500$ MeV, where weak-coupling QCD is not applicable, the description of color superconductivity is more adequate in the framework of effective theories for the low energy QCD region. In particular, since massless excitations might play an important role in different transport phenomena, a chiral effective meson theory for the pseudoscalar NG bosons of the CFL phase was also constructed [7,8] (see also the recent review on this topic [9] and references therein).

Another effective quark model approach to low density QCD is based on the Nambu–Jona-Lasinio (NJL) type of models. Since the NJL model contains the microscopic quark degrees of freedom, it is more fundamental and preferable, especially for the investigation of dynamical processes in dense baryonic matter, than chiral meson theories. On the other hand, it is also more suitable for the description of physics at low baryon densities than weak-coupling QCD. In particular, in the three-flavor NJL model the CFL effect was already considered, e.g., in [10] (see also the review [11]), where some aspects of the phase structure of dense quark matter were discussed, including the influence of the s-quark bare mass, color- and electric charge neutrality conditions, external magnetic field, etc. However, in spite of the fact that the lightest bosons may play an essential role e.g. in the cooling processes of neutron stars, up to now only few attention was paid to the consideration of the CFL ground state bosonic excitations, i.e. mesons and diquarks, and their dynamics in the framework of NJL models (see, however, the papers [12,13], where the properties of the massless NG boson, corresponding to the spontaneous breaking of the baryon $\mathrm{U}(1)_B$ symmetry in the CFL phase, were considered). In contrast, the properties of π-mesons and diquarks, surrounded by color superconducting quark matter, were already discussed in the framework of the two-flavor NJL model [14–18].

In the present paper we are going to study meson and diquark excitations of the CFL ground state in the framework of the massless three-flavor NJL model. As in [14–16], our consideration is based on the effective action, which is a generating functional for one-particle irreducible Green functions. They permit to get informations about the masses of bosonic excitations of the CFL phase. In the first step we will rederive the well–known result that in the normal phase of quark matter the NG bosons are just pseudoscalar mesons. Next, it will be shown that in the CFL phase the NG bosons are scalar- and pseudoscalar diquark excitations. Finally, we will demonstrate that in the CFL phase there appear massive scalar and pseudoscalar diquarks which are composed into a charged triplet and singlet as well as a neutral singlet of the SU(3) group.

II. NJL MODEL AND EFFECTIVE MESON-DIQUARK ACTION

Let us consider the following NJL model with three massless quark flavors

$$L = \bar{q}[\gamma^\nu i\partial_\nu + \mu\gamma^0]q + G_1 \sum_{a=0}^{8}[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma^5\tau_a q)^2] + G_2 \sum_{A=2,5,7}\sum_{A'=2,5,7}\{[\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q][\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C] + [\bar{q}^C\tau_A\lambda_{A'}q][\bar{q}\tau_A\lambda_{A'}q^C]\}. \quad (1)$$

In (1), $\mu \geq 0$ is the quark number chemical potential, which is the same for all quark flavors, $q^C = C\bar{q}^t$, $\bar{q}^C = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). The quark field $q \equiv q_{i\alpha}$ is a flavor and color triplet as well as a four-component Dirac spinor, where i, $\alpha = 1, 2, 3$. (Roman and Greek indices refer to flavor and color indices, respectively; spinor indices are omitted.) Furthermore, we use the notations τ_a, λ_a for Gell-Mann matrices in the flavor and color space, respectively, ($a = 1, \ldots, 8$); $\tau_0 = \sqrt{\frac{2}{3}}$ is proportional to the unit matrix in the flavor space. Clearly, the Lagrangian (1) as a whole is invariant under transformations from the color group $SU(3)_c$. In addition, it is symmetric under the chiral group $SU(3)_L \times SU(3)_R$ (chiral transformations act on the flavor indices of quark fields only) as well as under the baryon-number conservation group $U(1)_B$ and the axial group $U(1)_A$.[1]

The linearized version of the Lagrangian (1) contains collective bosonic fields and looks like

$$\tilde{L} = \bar{q}[\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma_a\tau_a - i\gamma^5\pi_a\tau_a]q - \frac{1}{4G_1}[\sigma_a\sigma_a + \pi_a\pi_a] - \frac{1}{4G_2}[\Delta^{s*}_{AA'}\Delta^s_{AA'} + \Delta^{p*}_{AA'}\Delta^p_{AA'}] - \frac{\Delta^{s*}_{AA'}}{2}[\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q] - \frac{\Delta^s_{AA'}}{2}[\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C] - \frac{\Delta^{p*}_{AA'}}{2}[\bar{q}^C\tau_A\lambda_{A'}q] - \frac{\Delta^p_{AA'}}{2}[\bar{q}\tau_A\lambda_{A'}q^C], \quad (2)$$

where here as well as in the following the summation over repeated indices $a = 0, \ldots, 8$ and A, $A' = 2, 5, 7$ is implied. Lagrangians (1) and (2) are equivalent which simply follows from the equations of motion for the bosonic fields

$$\begin{aligned}
\sigma_a(x) &= -2G_1(\bar{q}\tau_a q),\\
\Delta^s_{AA'}(x) &= -2G_2(\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q),\\
\Delta^{s*}_{AA'}(x) &= -2G_2(\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C),\\
\pi_a(x) &= -2G_1(\bar{q}i\gamma^5\tau_a q),\\
\Delta^p_{AA'}(x) &= -2G_2(\bar{q}^C\tau_A\lambda_{A'}q),\\
\Delta^{p*}_{AA'}(x) &= -2G_2(\bar{q}\tau_A\lambda_{A'}q^C).
\end{aligned} \quad (3)$$

One can easily see from (3) that the mesonic fields $\sigma_a(x)$, $\pi_a(x)$ are real quantities, i.e. $(\sigma_a(x))^\dagger = \sigma_a(x)$, $(\pi_a(x))^\dagger = \pi_a(x)$ (the superscript symbol $\dagger$ denotes the hermitian conjugation), whereas all diquark fields $\Delta^{s,p}_{AA'}(x)$ are complex ones, i.e.

$$(\Delta^s_{AA'}(x))^\dagger = \Delta^{s*}_{AA'}(x), \qquad (\Delta^p_{AA'}(x))^\dagger = \Delta^{p*}_{AA'}(x).$$

Moreover, $\Delta^s_{AA'}(x)$ and $\Delta^p_{AA'}(x)$ are scalars and pseudoscalars, correspondingly.

Let us introduce the flavor group $SU(3)_f = SU(3)_{L+R}$, which is the diagonal subgroup of the chiral group. Then, all scalar diquarks $\Delta^s_{AA'}(x)$ form an $(\bar{3}_c, \bar{3}_f)$-multiplet of the $SU(3)_c \times SU(3)_f$ group, i.e. they are a color and flavor antitriplet. The same is true for pseudoscalar diquarks $\Delta^p_{AA'}(x)$ which are also the components of an $(\bar{3}_c, \bar{3}_f)$-multiplet of the $SU(3)_c \times SU(3)_f$ group. Evidently, all diquarks $\Delta^{s,p}_{AA'}(x)$ have the same nonzero baryon charge. All the real $\sigma_a(x)$ and $\pi_a(x)$ fields are color singlets. Moreover, the set of scalar $\sigma_a(x)$-mesons is decomposed into a direct sum of the singlet and octet representations of the diagonal flavor group $SU(3)_f$. The same decomposition into multiplets is true for the set of all pseudoscalar $\pi_a(x)$-mesons. Clearly, in this case the octet is constructed from three pions ($\pi^\pm$ and π^0), four kaons (K^0, $\bar{K}^0$ and $K^\pm$) and the eta-meson (η_8), whereas the singlet (η_0) corresponds to the η'-meson.

Next, in order to use the Nambu–Gorkov formalism, we put the quark fields and their charge conjugates together into a bispinor

$$\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}$$

so that the Lagrangian (2) takes the compact form

$$\tilde{L} = -\frac{1}{4G_1}[\sigma_a\sigma_a + \pi_a\pi_a] - \frac{1}{4G_2}[\Delta^{s*}_{AA'}\Delta^s_{AA'} + \Delta^{p*}_{AA'}\Delta^p_{AA'}] + \frac{1}{2}\bar{\Psi}Z\Psi, \quad (4)$$

where Z is the 2×2-matrix in the Nambu–Gorkov space,

$$Z = \begin{pmatrix} D^+, & -K \\ -K^*, & D^- \end{pmatrix}, \quad (5)$$

and the following notations are adopted

[1] In a more realistic case, the additional 't Hooft six-quark interaction term should be taken into account in order to break the axial $U(1)_A$ symmetry [10]. However, in the present consideration we omit the 't Hooft term, for simplicity.

$$D^+ = i\gamma^\nu\partial_\nu + \mu\gamma^0 - \Sigma, \qquad \Sigma = \tau_a\sigma_a + i\gamma^5\pi_a\tau_a,$$
$$K = (\Delta^p_{AA'} + i\Delta^s_{AA'}\gamma^5)\tau_A\lambda_{A'}, \qquad D^- = i\gamma^\nu\partial_\nu - \mu\gamma^0 - \Sigma^t,$$
$$\Sigma^t = \tau^t_a\sigma_a + i\gamma^5\pi_a\tau^t_a, \qquad K^* = (\Delta^{p*}_{AA'} + i\Delta^{s*}_{AA'}\gamma^5)\tau_A\lambda_{A'}. \tag{6}$$

Now, integrating out the quark fields in the partition function with the Lagrangian (4) (this procedure is presented in detail in [16]), we obtain the effective action for the considered NJL model in the one-fermion loop approximation,

$$\mathcal{S}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = -\int d^4x\left[\frac{\sigma_a^2+\pi_a^2}{4G_1} + \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2}\right] - \frac{i}{2}\,{\rm Tr}\ln Z. \tag{7}$$

Besides of an evident trace over the two-dimensional Nambu–Gorkov matrix, the Tr-operation in (7) stands for calculating the trace in spinor, flavor, color as well as four-dimensional coordinate spaces, correspondingly. Let us suppose that parity is conserved so that all pseudoscalar diquark and meson fields have zero ground state expectation values, i.e. $\langle\Delta^p_{AA'}(x)\rangle = 0$ and $\langle\pi_a(x)\rangle = 0$. Furthermore, since at zero s-quark mass, $m_s = 0$, only the competition between the normal quark matter phase and the CFL one is relevant to the physics of dense QCD (see, e.g., Fig. 1 in [2], where the corresponding phase portrait for QCD with two massless u, d-quarks at zero temperature is presented in terms of μ, m_s), we permit in the present consideration nonzero ground state expectation values only for $\sigma_0(x)$ and $\Delta^s_{AA}(x)$ fields ($A = 2, 5, 7$). Namely, let $\langle\sigma_0(x)\rangle \equiv \sigma^o_0$, $\langle\Delta^s_{AA}(x)\rangle \equiv \Delta$, $\langle\Delta^{s*}_{AA}(x)\rangle \equiv \Delta^*$, where $A = 2, 5, 7$, but other boson fields from (3) have zero ground state expectation values. In this case, if $\Delta = 0$, then quark matter is in the normal phase, where at $\sigma^o_0 \neq 0$ the ground state is an $SU(3)_c \times SU(3)_f \times U(1)_B$-invariant one. If $\Delta \neq 0$, then the CFL phase is realized in the model, and the initial symmetry is spontaneously broken down to $SU(3)_{L+R+c}$. Now, let us make the following shifts of bosonic fields in (7): $\sigma_0(x) \to \sigma_0(x) + \sigma^o_0$, $\Delta^{s*}_{AA}(x) \to \Delta^{s*}_{AA}(x) + \Delta^*$, $\Delta^s_{AA}(x) \to \Delta^s_{AA}(x) + \Delta$, where $A = 2, 5, 7$, and other bosonic fields remain unshifted. (Obviously, the new shifted bosonic fields $\sigma_0(x)$, $\Delta^s_{AA}(x)$, etc. now denote the (small) quantum fluctuations around the mean values σ^o_0, Δ, etc. of mesons and diquarks rather than the original fields (3).) In this case

$$Z \to \begin{pmatrix} D^+_o, & -K_o \\ -K^*_o, & D^-_o \end{pmatrix} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix} \equiv S_0^{-1} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}, \tag{8}$$

where K_o, K^*_o, $D^\pm_o$, Σ_o, Σ^t_o are the corresponding quantities (6), in which all bosonic fields are replaced by their own ground state expectation values, i.e. $\sigma_0(x) \to \sigma^o_0$, $\pi_a(x) \to 0$, $\Delta^s_{AA}(x) \to \Delta$, $\Delta^p_{AA'}(x) \to 0$, etc. and S_0 is the quark propagator matrix in the Nambu–Gorkov representation (its matrix elements S_{ij} are given in the Appendix). Then, expanding the obtained expression into a Taylor-series up to second order of small bosonic fluctuations, we have

$$\mathcal{S}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = \mathcal{S}^{(0)}_{\rm eff} + \mathcal{S}^{(2)}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) + \cdots, \tag{9}$$

where

$$\mathcal{S}^{(0)}_{\rm eff} = -\int d^4x\left[\frac{\sigma^o_0\sigma^o_0}{4G_1} + \frac{3|\Delta|^2}{4G_2}\right] + \frac{i}{2}\,{\rm Tr}\ln(S_0) \equiv -\Omega(\sigma^o_0, \Delta, \Delta^*)\int d^4x, \tag{10}$$

$$\mathcal{S}^{(2)}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = -\int d^4x\left[\frac{\sigma_a^2+\pi_a^2}{4G_1} + \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2}\right] + \frac{i}{4}\,{\rm Tr}\left\{S_0\begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}S_0\begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}\right\}. \tag{11}$$

Notice that the term linear in meson and diquark fields vanishes in (9) due to the gap equations (see below). In (10) the quantity Ω is the thermodynamic potential of the system. In terms of $M = \sqrt{\frac{2}{3}}\sigma^o_0$ it looks like

$$\Omega(M, \Delta, \Delta^*) = \frac{3M^2}{8G_1} + \frac{3|\Delta|^2}{4G_2} - 8\int\frac{d^3q}{(2\pi)^3}\{E^+_\Delta + E^-_\Delta\} - \int\frac{d^3q}{(2\pi)^3}\{E^+_{2\Delta} + E^-_{2\Delta}\}, \tag{12}$$

where $(E^\pm_\Delta)^2 = (E^\pm)^2 + |\Delta|^2$, $(E^\pm_{2\Delta})^2 = (E^\pm)^2 + 4|\Delta|^2$, $E^\pm = E \pm \mu$, $E = \sqrt{\vec{q}^2 + M^2}$. Starting from (12), one can find the gap equations $\partial\Omega/\partial\Delta^* = 0$ and $\partial\Omega/\partial M = 0$, which supply us with the values of M, Δ in the ground state of the system:

$$\frac{\partial\Omega}{\partial M} \equiv \frac{3M}{4G_1} - 2M\int\frac{d^3q}{(2\pi)^3E}\left\{\frac{4E^+}{E^+_\Delta} + \frac{4E^-}{E^-_\Delta} + \frac{E^+}{2E^+_{2\Delta}} + \frac{E^-}{2E^-_{2\Delta}}\right\} = 0, \tag{13}$$

$$\frac{\partial\Omega}{\partial\Delta^*} \equiv \frac{3\Delta}{4G_2} - \Delta\int\frac{d^3q}{(2\pi)^3}\left\{\frac{4}{E^+_\Delta} + \frac{4}{E^-_\Delta} + \frac{2}{E^+_{2\Delta}} + \frac{2}{E^-_{2\Delta}}\right\} = 0. \tag{14}$$

Since the integrals in the right hand sides of (12)–(14) are ultraviolet divergent, we regularize them as well as the other three-dimensional divergent integrals below by implementing a cutoff in the integration regions, $|\vec{q}| < \Lambda$. In all numerical calculations below, we use the following values of the model parameters (see, e.g., Ref. [11])

$$\Lambda = 602.3 \text{ MeV}, \qquad G_1\Lambda^2 = 2.319, \qquad G_2 = 3G_1/4. \tag{15}$$

Suppose that Δ is a real quantity. In this case, since the thermodynamic potential Ω (12) is an even function in both the M and Δ variables, it is enough to study it only in the region $\{M \geq 0, \Delta \geq 0\}$. Just for this region a graph of $\Omega(M, \Delta)$ is presented in Fig. 1 at $\mu = \mu_c \approx 329$ MeV. It is clear from Fig. 1 that in this case the thermodynamic potential has two global minimum points (GMP), A and B. The point A with coordinates $(M_c, 0)$, where $M_c \approx 355$ MeV, lies on the M-axis and corresponds to the normal quark matter phase, whereas the point B with coordinates $(0, \Delta_c)$, $\Delta_c \approx 86$ MeV, is arranged on the Δ-axis and corresponds to the CFL phase of dense baryonic matter. (Evidently, both A and B are solutions of the gap Eqs. (13) and (14). In addition, as is seen from Fig. 1, there are another solutions of the gap equations which, however, correspond to a maximum- or saddle points of the thermodynamic potential.) If $\mu < \mu_c$, the function Ω has only one GMP of the A-type, i.e. it lies on the M-axis. So, in this case the normal quark matter phase is realized in the model. In contrast, if $\mu > \mu_c$, then the single GMP of the Ω-function is located on the Δ-axis, i.e. it is of the B-type, and the CFL phase occurs. The dependence of the coordinates (gaps) of the GMP on the chemical potential is presented in Fig. 2. Since the GMP jumps at $\mu = \mu_c$ from A to B (or vice versa), one may conclude that at $\mu = \mu_c$ a first order phase transition takes place in the system.

The effective action $\mathcal{S}_{\text{eff}}^{(2)}$ in (11) is really a generating functional of the one-particle irreducible (1PI) two-point Green functions of mesons and diquarks both in the normal and CFL phases, namely

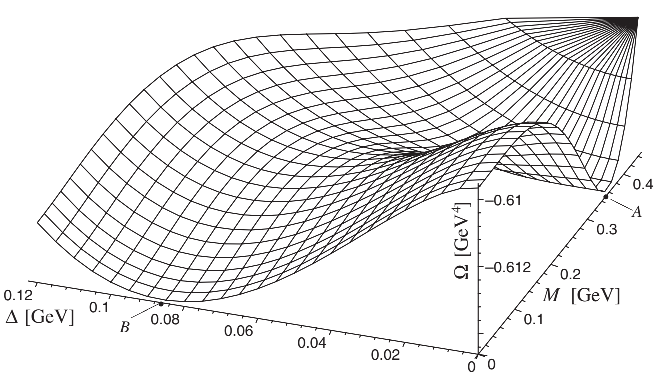

FIG. 1. The behavior of the thermodynamic potential Ω vs M and Δ at the critical value of the chemical potential $\mu_c \approx 329$ MeV.

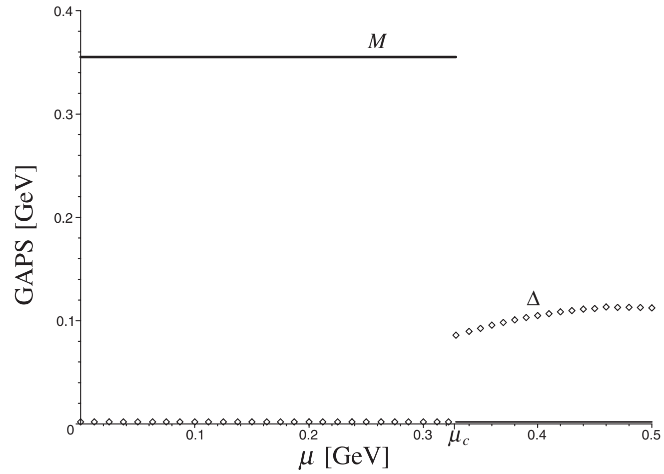

FIG. 2. The coordinates M and Δ (gaps) of the global minimum point of the thermodynamic potential vs the chemical potential μ. Here $\mu_c \approx 329$ MeV.

$$\Gamma_{XY}(x - y) = -\frac{\delta^2 \mathcal{S}_{\text{eff}}^{(2)}}{\delta Y(y)\delta X(x)}, \tag{16}$$

where $X(x), Y(x) = \sigma_a(x), \pi_b(x), \Delta_{AA'}^{s,p}(x), \Delta_{BB'}^{s,p*}(x)$. These Green functions are very useful, in particular, in determining the dispersion relations and masses of particles. In the following, we shall say that in the theory there is a *mixing* between two different particles with corresponding fields $X(x)$ and $Y(x)$, if their 1PI Green function $\Gamma_{XY}(x - y)$ is not equal to zero. By analyzing the structure of the effective action (11), it is possible to show that in the NJL model (1) with three massless quarks, mesons and diquarks are not mixed both in the normal ($\mu < \mu_c$) and CFL phases ($\mu > \mu_c$).[2] Moreover, each pseudoscalar $\pi_a(x)$ field (as well as scalar $\sigma_b(x)$ one) does not mix with other meson fields, apart from itself. So, in the normal phase (where $\Delta = 0$ and $M \approx 355$ MeV) one can obtain the following expression $\bar{\Gamma}_{\pi_i\pi_i}(p)$ for the Fourier transformation of the two-point 1PI Green function of pseudoscalar mesons $\pi_i(x)$, taken in the rest frame $p = (p_0, 0, 0, 0)$:

$$\bar{\Gamma}_{\pi_i\pi_i}(p_0) = p_0^2 \int \frac{d^3q}{(2\pi)^3} \cdot \frac{12}{E[p_0^2 - 4E^2]}. \tag{17}$$

(In obtaining (17), the gap Eq. (13) was used in order to eliminate the coupling constant G_1 from the final expression.) Evidently, this expression turns into zero at $p_0^2 = 0$. Since the relation (17) is true for $i = 0, 1, \ldots, 8$, it means that nine massless excitations, Nambu–Goldstone (NG)

[2]Note, if some of the current quark masses are nonzero, then there arises a mixing between mesons and diquarks in the CFL phase. This effect is the analogy of the mixing between the σ-meson and the scalar diquark in the color superconducting phase of a two-flavor NJL model with nonzero masses of u- and d-quarks [15,16].

bosons, do exist in the pseudoscalar meson sector of the model in the normal phase. This fact corresponds to a spontaneous symmetry breaking in the normal phase down to the group $SU(3)_c \times SU(3)_f \times U(1)_B$.

Recall, that our initial NJL model (1) is invariant under the symmetry group $SU(3)_L \times SU(3)_R \times SU(3)_c \times U(1)_B \times U(1)_A$, which contains an additional (nonphysical) axial subgroup $U(1)_A$. So in the CFL phase, where the ground state is symmetric under the group $SU(3)_{L+R+c}$, an additional NG boson must appear in the mass spectrum of the model giving altogether 18 NG bosons.

Note that in the sector of the meson excitations of the CFL phase we do not find any massless particle.

III. MASSLESS DIQUARK EXCITATIONS IN THE CFL PHASE

A further consideration of the effective action (11) shows that in the CFL phase, where $M = 0$ and $\Delta \neq 0$, scalar and pseudoscalar diquarks are separated from each other. Moreover, the 18 pseudoscalar diquarks (nine $\Delta^p_{AA'}(x)$ and nine $\Delta^{p*}_{AA'}(x)$ fields) may be divided into four sectors: $p(57, 75)$, $p(25, 52)$, $p(27, 72)$ and $p(257)$. Each of the sectors $p(AA', A'A)$, where $A \neq A'$, is composed of $\Delta^p_{AA'}(x)$, $\Delta^{p*}_{AA'}(x)$, $\Delta^p_{A'A}(x)$ and $\Delta^{p*}_{A'A}(x)$-diquarks, whereas the sector $p(257)$ is composed of six fields, $\Delta^{p*}_{22}(x)$, $\Delta^{p*}_{55}(x)$, $\Delta^{p*}_{77}(x)$, $\Delta^p_{22}(x)$, $\Delta^p_{55}(x)$, and $\Delta^p_{77}(x)$. There is a mixing between diquarks from the same sector, however fields from different sectors are not mixed. (The analogous situation takes place for the set of scalar diquarks.)

Partially, these mixing properties of the pseudoscalar diquarks are explained by the ground state $SU(3)_{L+R+c}$ symmetry of the CFL phase. Indeed, with respect to this group all pseudoscalar diquarks are decomposed into a direct sum of the $\bar{3}$ and $\bar{6}$ multiplets. The fields $\Delta^p_{AA}(x)$ with $A = 2, 5, 7$ are in an antitriplet $\bar{3}$, whereas all diquarks of the form $\Delta^p_{A'A}(x)$ $(A' \neq A)$ are in an antisixtet $\bar{6}$. It is clear from symmetry considerations that diquarks from $\bar{3}$ do not mix with diquarks from $\bar{6}$ (here we use the terminology, introduced just after (16)). Since the mixing between a particle and corresponding antiparticle is allowed as a rule, we may conclude that pseudoscalar diquarks from the sector $p(257)$ are separated from the other six $\Delta^p_{A'A}(x)$- and six $\Delta^{p*}_{A'A}(x)$-diquarks $(A' \neq A)$. A further separation between components of the $\bar{6}$-multiplet and corresponding antiparticles occurs on a dynamical basis, i.e. it is due to the structure of the effective action (11) and the quark propagator matrix (A1)–(A4). As a result, one can show that these diquarks are divided into the three above-mentioned sectors, $p(57, 75)$, $p(25, 52)$, $p(27, 72)$.

Below we suppose that the gap Δ is a real positive number in the CFL phase.

A. The case of $p(AA', A'A)$ sectors

Let us first study the mass spectrum of the excitations, e.g., in the sector $p(57, 75)$. The two-point 1PI Green functions of pseudoscalar diquarks from this sector can be obtained from (11) by the relation (16). In the rest frame, where $p = (p_0, 0, 0, 0)$, the Fourier transforms of these 1PI Green functions form the following matrix:

$$\bar{\Gamma}_{57,75}(p_0) = \begin{pmatrix} \bar{\Gamma}_{\Delta^p_{57}\Delta^p_{57}}(p_0) & \bar{\Gamma}_{\Delta^p_{57}\Delta^{p*}_{57}}(p_0) & \bar{\Gamma}_{\Delta^p_{57}\Delta^p_{75}}(p_0) & \bar{\Gamma}_{\Delta^p_{57}\Delta^{p*}_{75}}(p_0) \\ \bar{\Gamma}_{\Delta^{p*}_{57}\Delta^p_{57}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{57}\Delta^{p*}_{57}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{57}\Delta^p_{75}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{57}\Delta^{p*}_{75}}(p_0) \\ \bar{\Gamma}_{\Delta^p_{75}\Delta^p_{57}}(p_0) & \bar{\Gamma}_{\Delta^p_{75}\Delta^{p*}_{57}}(p_0) & \bar{\Gamma}_{\Delta^p_{75}\Delta^p_{75}}(p_0) & \bar{\Gamma}_{\Delta^p_{75}\Delta^{p*}_{75}}(p_0) \\ \bar{\Gamma}_{\Delta^{p*}_{75}\Delta^p_{57}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{75}\Delta^{p*}_{57}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{75}\Delta^p_{75}}(p_0) & \bar{\Gamma}_{\Delta^{p*}_{75}\Delta^{p*}_{75}}(p_0) \end{pmatrix}. \quad (18)$$

After tedious but straight-forward calculations, based on the technique elaborated in our previous papers [14–16] (see also [17,18]) and used in the consideration of mesons and diquarks in the color superconducting phase of the two-flavor NJL model, the matrix (18) takes the form:

$$\bar{\Gamma}_{57,75}(p_0) = \begin{pmatrix} 0 & A & C & 0 \\ B & 0 & 0 & C \\ C & 0 & 0 & A \\ 0 & C & B & 0 \end{pmatrix}, \quad (19)$$

where $A \equiv \alpha + p_0\beta$, $B \equiv \alpha - p_0\beta$ and

$$\alpha = \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{6E^+_\Delta p_0^2 - (E^+_\Delta + E^+_{2\Delta})^2(2E^+_\Delta + E^+_{2\Delta})}{9E^+_\Delta E^+_{2\Delta}[p_0^2 - (E^+_\Delta + E^+_{2\Delta})^2]} + \frac{4p_0^2 + 4(E^+_\Delta)^2 - 10\Delta^2}{3E^+_\Delta[p_0^2 - 4(E^+_\Delta)^2]} \right\} + \int \frac{d^3q}{(2\pi)^3} \{E^+_\Delta \to E^-_\Delta, E^+_{2\Delta} \to E^-_{2\Delta}\}, \quad (20)$$

$$\beta = \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{E^+(E_\Delta^+ + E_{2\Delta}^+)}{3E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]} + \frac{10E^+}{3E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} \right\} - \int \frac{d^3q}{(2\pi)^3} \{E^+ \to E^-, E_\Delta^+ \to E_\Delta^-, E_{2\Delta}^+ \to E_{2\Delta}^-\}, \quad (21)$$

$$C = \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{2\Delta^2(E_\Delta^+ + E_{2\Delta}^+)}{3E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]} - \frac{10\Delta^2}{3E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} \right\} + \int \frac{d^3q}{(2\pi)^3} \{E_\Delta^+ \to E_\Delta^-, E_{2\Delta}^+ \to E_{2\Delta}^-\}. \quad (22)$$

(To obtain the above expressions for α and β, we have used the gap Eq. (14) in order to eliminate the coupling constant G_2 from corresponding 1PI Green functions.) Evidently, in the case of mixing between particles the information about their masses should be extracted from zeros of a matrix determinant, composed from corresponding 1PI Green functions. So, in our case it is necessary to study the equation $\det\bar{\Gamma}_{57,75}(p_0) = 0$, which takes the following form

$$\det\bar{\Gamma}_{57,75}(p_0) \equiv (AB - C^2)^2 = [(\alpha - C)(\alpha + C) - p_0^2\beta^2]^2 = 0. \quad (23)$$

In the p_0^2-plane, each zero of this equation defines a mass squared of a bosonic excitation of the CFL phase ground state in the $p(57, 75)$ sector. Since this sector contains four pseudoscalar diquarks, one should search for four solutions of the Eq. (23) in the p_0^2-plane. However, due to the structure of (23), it is clear that this equation admits at least two different solutions (each being two-fold degenerate), which are given by the zeros of the expression in the square bracket. Since

$$\alpha - C = p_0^2 \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{p_0^2[2E_\Delta^+ + 4E_{2\Delta}^+] - 8(E_\Delta^+)^3 - (E_\Delta^+)^2 E_{2\Delta}^+ - 10E_\Delta^+(E_{2\Delta}^+)^2 - 5(E_{2\Delta}^+)^3}{3E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2][p_0^2 - 4(E_\Delta^+)^2]} \right\} + p_0^2 \int \frac{d^3q}{(2\pi)^3} \{E_\Delta^+ \to E_\Delta^-, E_{2\Delta}^+ \to E_{2\Delta}^-\} \equiv p_0^2 F(p_0^2), \quad (24)$$

the square bracket in (23) becomes zero at the point $p_0^2 = 0$. So, in the $p(57, 75)$ sector there are two massless pseudoscalar excitations, i.e. NG bosons. Two other excitations have the same nontrivial mass squared which is the solution of the equation

$$(\alpha + C)F(p_0^2) - \beta^2 = 0. \quad (25)$$

Its investigation is outside the scope of the present paper. A similar situation occurs in the other four-component sectors $p(25, 52)$ and $p(27, 72)$. Namely, for both sectors the 1PI Green function matrix has the form (19). Hence, in each of these sectors there are two NG bosons as well as two massive excitations with the same mass squared. Its value is given by the solution of the Eq. (25).

To summarize, in the pseudoscalar diquark sectors $p(57, 75)$, $p(25, 52)$, and $p(27, 72)$ we have found six massless diquark excitations, which are NG bosons, as well as six pseudoscalar diquark excitations with a nonzero mass which is a solution of the Eq. (25). In total, these six massive real pseudoscalar diquarks form a complex (charged) triplet of the $SU(3)_{L+R+c}$ group.

B. The case of the $p(257)$ sector

All the two-point 1PI Green functions of the pseudoscalar diquarks, entering the $p(257)$ sector, are defined by the relation (16). In momentum space representation and in the rest frame, i.e. at $p = (p_0, 0, 0, 0)$, they form the following 6×6 matrix:

$$\bar{\Gamma}_{257}(p_0) = \begin{pmatrix} \bar{\Gamma}_{\Delta_{22}^p\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^p\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{22}^p\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^p\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{22}^p\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^p\Delta_{77}^{p*}}(p_0) \\ \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{22}^{p*}\Delta_{77}^{p*}}(p_0) \\ \bar{\Gamma}_{\Delta_{55}^p\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^p\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{55}^p\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^p\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{55}^p\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^p\Delta_{77}^{p*}}(p_0) \\ \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{55}^{p*}\Delta_{77}^{p*}}(p_0) \\ \bar{\Gamma}_{\Delta_{77}^p\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^p\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{77}^p\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^p\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{77}^p\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^p\Delta_{77}^{p*}}(p_0) \\ \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{22}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{22}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{55}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{55}^{p*}}(p_0) & \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{77}^p}(p_0) & \bar{\Gamma}_{\Delta_{77}^{p*}\Delta_{77}^{p*}}(p_0) \end{pmatrix}. \quad (26)$$

A tedious but straight-forward calculation yields:

$$\bar{\Gamma}_{257}(p_0) = \begin{pmatrix} R & P & T & W & T & W \\ Q & R & Z & T & Z & T \\ T & W & R & P & T & W \\ Z & T & Q & R & Z & T \\ T & W & T & W & R & P \\ Z & T & Z & T & Q & R \end{pmatrix}. \qquad (27)$$

The determinant of this matrix looks like

$$\det\bar{\Gamma}_{257}(p_0) = [(T-R)^2 - (W-P)(Z-Q)]^2 \times \{(2T+R)^2 - (2W+P)(2Z+Q)\}, \qquad (28)$$

where $P = I_0 + p_0 I_1$, $Q = I_0 - p_0 I_1$, $W = J_0 + p_0 J_1$, $Z = J_0 - p_0 J_1$, and

$$I_0(p_0^2) = \frac{1}{4G_2} + \int \frac{d^3q}{(2\pi)^3}\left\{\frac{28}{9}\cdot\frac{(E_\Delta^+)^2 + (E^+)^2}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} + \frac{4}{9}\cdot\frac{(E_{2\Delta}^+)^2 + (E^+)^2}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} + \frac{2}{9}\cdot\frac{(E_\Delta^+ + E_{2\Delta}^+)[E_\Delta^+ E_{2\Delta}^+ + (E^+)^2]}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-, E^+ \to E^-\}, \qquad (29)$$

$$I_1(p_0^2) = \int \frac{d^3q}{(2\pi)^3}\left\{\frac{28}{9}\cdot\frac{E^+}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} + \frac{4}{9}\cdot\frac{E^+}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} + \frac{2}{9}\cdot\frac{(E_\Delta^+ + E_{2\Delta}^+)E^+}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-, E^+ \to -E^-\}, \qquad (30)$$

$$R(p_0^2) = (-\Delta^2)\int \frac{d^3q}{(2\pi)^3}\left\{\frac{28}{9}\cdot\frac{1}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} + \frac{16}{9}\cdot\frac{1}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} - \frac{4}{9}\cdot\frac{E_\Delta^+ + E_{2\Delta}^+}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-\}, \qquad (31)$$

$$J_0(p_0^2) = \int \frac{d^3q}{(2\pi)^3}\left\{\frac{4}{9}\cdot\frac{(E_{2\Delta}^+)^2 + (E^+)^2}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} - \frac{2}{9}\cdot\frac{(E_\Delta^+)^2 + (E^+)^2}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} - \frac{1}{9}\cdot\frac{(E_\Delta^+ + E_{2\Delta}^+)[E_\Delta^+ E_{2\Delta}^+ + (E^+)^2]}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-, E^+ \to E^-\}, \qquad (32)$$

$$J_1(p_0^2) = \int \frac{d^3q}{(2\pi)^3}\left\{\frac{4}{9}\cdot\frac{E^+}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} - \frac{2}{9}\cdot\frac{E^+}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} - \frac{1}{9}\cdot\frac{(E_\Delta^+ + E_{2\Delta}^+)E^+}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-, E^+ \to -E^-\}, \qquad (33)$$

$$T(p_0^2) = (-\Delta^2)\int \frac{d^3q}{(2\pi)^3}\left\{\frac{16}{9}\cdot\frac{1}{E_{2\Delta}^+[p_0^2 - 4(E_{2\Delta}^+)^2]} - \frac{2}{9}\cdot\frac{1}{E_\Delta^+[p_0^2 - 4(E_\Delta^+)^2]} + \frac{2}{9}\cdot\frac{E_\Delta^+ + E_{2\Delta}^+}{E_\Delta^+ E_{2\Delta}^+[p_0^2 - (E_\Delta^+ + E_{2\Delta}^+)^2]}\right\} + \int \frac{d^3q}{(2\pi)^3}\{E_{\Delta,2\Delta}^+ \to E_{\Delta,2\Delta}^-\}. \qquad (34)$$

In terms of the I_k, J_l-functions, the expression (28) can be rewritten as

$$\det\bar{\Gamma}_{257}(p_0) = [(T - R - J_0 + I_0)(T - R + J_0 - I_0) + p_0^2(J_1 - I_1)^2]^2\{(2T + R - 2J_0 - I_0)(2T + R + 2J_0 + I_0) + p_0^2(2J_1 + I_1)^2\}. \qquad (35)$$

As in the previous section, the full spectrum of the CFL phase excitations in the $p(257)$-sector of the model can be explicitly computed by evaluating the zeros of the determinant (28) (or (35), alternatively). Since the $p(257)$-sector consists of six diquark degrees of freedom, we expect that $\det\bar{\Gamma}_{257}(p_0)$ has at least six zeros in the p_0^2-plane.

Eliminating in (29) the coupling constant G_2 again with the help of the gap Eq. (14), one can easily show that $T(0) = J_0(0)$ and $R(0) = I_0(0)$. Evidently, in this case both the square bracket expression and the brace one from (35) turn into zero at $p_0^2 = 0$. Hence, in the $p(257)$-sector of the model there are three pseudoscalar NG boson excitations. Now let us obtain some information about the remaining three excitations, which are massive ones. For this purpose, similarly to (24), it is possible to extract in an evident form the factor p_0^2 from the expressions in the curly brackets of (35), i.e.

$$T - R - J_0 + I_0 \equiv p_0^2\phi(p_0^2),$$
$$2T + R - 2J_0 - I_0 \equiv p_0^2\varphi(p_0^2). \tag{36}$$

Then it is clear from (35) that two of these excitations have an identical mass which is a solution of the equation

$$\phi(p_0^2)(T - R + J_0 - I_0) + (J_1 - I_1)^2 = 0. \tag{37}$$

In total, they form a complex (charged) singlet of the group $SU(3)_{R+L+c}$. Finally, there is a further diquark excitation, whose mass obeys another equation

$$\varphi(p_0^2)(2T + R + 2J_0 + I_0) + (2J_1 + I_1)^2 = 0. \tag{38}$$

Evidently, this diquark is also a $SU(3)_{R+L+c}$-singlet. As a result, we have proved that there are nine pseudoscalar NG bosons (diquarks) in the mass spectrum of the initial NJL model. Moreover, it contains massive pseudoscalar diquarks, composed of a complex triplet and singlet as well as real singlet of the group $SU(3)_{f+c}$.

Since in the chiral limit, where all quarks are massless, the two-point 1PI Green functions of scalar diquarks are identical to the corresponding Green functions of pseudoscalar diquark fields, one may conclude that the diquark spectrum contains nine scalar massless excitations. In real QCD, eight of them should supply a mass to gluons through the Anderson–Higgs mechanism, but the remaining massless excitation is the NG boson corresponding to the spontaneous breaking of the baryon $U(1)_B$ symmetry. In addition, there are a massive complex triplet and singlet as well as a real singlet of scalar diquarks in the mass spectrum of the model.

IV. CONCLUSIONS

In the present paper the two-point 1PI Green functions of scalar and pseudoscalar diquarks are investigated in the framework of a three-flavor NJL model with massless quarks and nonzero chemical potential μ. The model contains interaction terms both in the quark-antiquark and quark-quark channels, but the ‘t Hooft six-quark term is omitted, for simplicity (see (1)). In this case, the initial symmetry group of the model, i.e. $SU(3)_L \times SU(3)_R \times SU(3)_c \times U(1)_B \times U(1)_A$ does contain the axial $U(1)_A$ subgroup. As a result, we have shown that at sufficiently low values of μ, $\mu < \mu_c \approx 330$ MeV, the normal quark matter phase with $SU(3)_{L+R} \times SU(3)_c \times U(1)_B$ is realized and nine massless pseudoscalar mesons (which are the NG bosons), $\pi^\pm$, π^0, K^0, $\bar{K}^0$, $K^\pm$, η_8 and η', appear. (In massless QCD, where $U(1)_A$ is broken on the quantum level, or in NJL models with ‘t Hooft interaction the η'-meson is not a NG boson.)

At $\mu > \mu_c$ the original symmetry of the model is spontaneously broken down to $SU(3)_{L+R+c}$, and the CFL phase does occur. In this case, in accordance with the Goldstone theorem, eighteen NG bosons must appear in the mass spectrum. Considering 1PI Green functions, we have found nine NG bosons in the sector of scalar diquark excitations. Eight of them have to be considered as nonphysical, since in real QCD they supply masses to gluons by the Anderson–Higgs mechanism. The remaining scalar NG boson corresponds to a spontaneous breaking of the baryon $U(1)_B$ symmetry. The other nine NG bosons are no more pseudoscalar mesons, but now the massless excitations in the pseudoscalar diquark sector of the model.

Besides NG diquarks, we have proved the existence of massive diquark excitations in the CFL phase. They form both pseudoscalar and scalar complex (charged) triplets and singlets as well as a real (neutral) singlet of the group $SU(3)_{f+c}$. There arises the interesting question, whether these diquark masses are above the threshold of fermion pair excitations so that massive diquarks might eventually decay into two NG bosons.[3] The detailed numerical investigation of diquark masses as functions of the chemical potential is outside the scope of this paper and will be considered in a future publication. There, we are going to study also the masses of mesons in the environment of dense quark matter in the CFL phase.

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APPENDIX

In the Nambu–Gorkov representation the inverse quark propagator matrix S_0^{-1} is given in (8). Using the techniques, elaborated in [14–18], it is possible to obtain the following expressions for the matrix elements of the quark propagator matrix

$$S_0 \equiv \begin{pmatrix} S_{11}, S_{12} \\ S_{21}, S_{22} \end{pmatrix}$$

[3]Our earlier investigations of the dispersion relations of diquarks in color superconducting quark matter, composed of u and d quarks (see the papers [15]), indicate that massive diquarks may occur as resonances which are heavily damped. However, the situation in the CFL phase might be different and needs a special consideration.

$$S_{11}(x,y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{q_0 - E^+}{q_0^2 - (E_{B\Delta}^+)^2} \gamma^0 \bar{\Lambda}_+ + \frac{q_0 + E^-}{q_0^2 - (E_{B\Delta}^-)^2} \gamma^0 \bar{\Lambda}_- \right\}, \quad (A1)$$

$$S_{12}(x,y) = -i\Delta B \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{1}{q_0^2 - (E_{B\Delta}^+)^2} \gamma^5 \bar{\Lambda}_- + \frac{1}{q_0^2 - (E_{B\Delta}^-)^2} \gamma^5 \bar{\Lambda}_+ \right\}, \quad (A2)$$

$$S_{21}(x,y) = -i\Delta^* B \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{1}{q_0^2 - (E_{B\Delta}^+)^2} \gamma^5 \bar{\Lambda}_+ + \frac{1}{q_0^2 - (E_{B\Delta}^-)^2} \gamma^5 \bar{\Lambda}_- \right\}, \quad (A3)$$

$$S_{22}(x,y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{q_0 + E^+}{q_0^2 - (E_{B\Delta}^+)^2} \gamma^0 \bar{\Lambda}_- + \frac{q_0 - E^-}{q_0^2 - (E_{B\Delta}^-)^2} \gamma^0 \bar{\Lambda}_+ \right\}, \quad (A4)$$

where $M = \sqrt{\frac{2}{3}}\sigma_0^o$, $\bar{\Lambda}_\pm = \frac{1}{2}(1 \pm \frac{\gamma^0(\vec{\gamma}\vec{q} - M)}{E})$. Moreover, $(E_{B\Delta}^\pm)^2 = (E^\pm)^2 + |\Delta|^2 B^2$, $E^\pm = E \pm \mu$, $E = \sqrt{\vec{q}^2 + M^2}$ and $B = \sum_{A=2,5,7} \tau_A \lambda_A$. (In these and other similar expressions, q_0 is a shorthand notation for $q_0 + i\varepsilon \cdot \mathrm{sgn}(q_0)$, where the limit $\varepsilon \to 0_+$ must be taken at the end of all calculations. This prescription correctly implements the role of μ as the chemical potential and preserves the causality of the theory.)

It is clear from (A1)–(A4) that all color- and flavor dependences in the matrix elements S_{11}, S_{12}, S_{21} and S_{22} arise only due to the matrix B. In the nine-dimensional space $c \times f$, which is the direct production of color and flavor spaces, the two 9×9 matrices B and B^2 take the following forms

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B^2 = \begin{pmatrix} 2, 0, 0, 0, 1, 0, 0, 0, 1 \\ 0, 1, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 1, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 1, 0, 0, 0, 0, 0 \\ 1, 0, 0, 0, 2, 0, 0, 0, 1 \\ 0, 0, 0, 0, 0, 1, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 1, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 1, 0 \\ 1, 0, 0, 0, 1, 0, 0, 0, 2 \end{pmatrix}. \quad (A5)$$

Thus, the quark propagator S_0 may be thought as the matrix in the $c \times f$ space as well. Since it is an infinite series in powers of B, the diagonalization of the matrix B means the diagonalization of the propagator S_0 in the $c \times f$ space. Let us define in the $c \times f$ space the following matrix (its rows are the ortonormal eigenvectors of the B-matrix, corresponding to the eigenvalues $\rho_1 = \cdots = \rho_5 = 1$, $\rho_6 = \rho_7 = \rho_8 = -1$, $\rho_9 = -2$):

$$O = \begin{pmatrix} \frac{1}{\sqrt{6}} & 0 & 0 & 0 & -\frac{2}{\sqrt{6}} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (A6)$$

Evidently, we have $OO^t = 1$, $\det O = 1$ and $OB^2O^t = \mathrm{diag}(1, 1, \ldots, 1, 4)$ $OBO^t = \mathrm{diag}(1, 1, 1, 1, 1, -1, -1, -1, -2)$. Moreover, the matrix O diagonalizes the quark propagator S_0 in the direct-product space $c \times f$: $OS_0O^t = \mathrm{diag}(S_1, \ldots, S_9)$, where each of $S_1, \ldots, S_8$ corresponds to a Nambu–Gorkov representation of a propagator of the quasiparticle with gap $|\Delta|$, whereas S_9—to a quasiparticle with gap $2|\Delta|$. Hence, in the CLF phase all nine quasiparticles form an SU(3)-octet with gap $|\Delta|$ and an SU(3)-singlet with gap $2|\Delta|$ (see also [5,11]).

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