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Mass spectrum of diquarks and mesons in the color–flavor locked phase of dense quark matter

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The spectrum of meson and diquark excitations of dense quark matter is considered in the framework of the Nambu – Jona-Lasinio model with three types of massless quarks in the presense of a quark number chemical potential μ. We investigate the effective action of meson- and diquark fields both at sufficiently large values of $\mu > \mu_c \approx 330$ MeV, where the color–flavor locked (CFL) phase is realized, and in the chirally broken phase of quark matter ($\mu < \mu_c$). In the last case all nine pseudoscalar mesons are Nambu – Goldstone (NG) bosons, whereas the mass of the scalar meson nonet is twice the dynamical quark mass. In the chirally broken phase the pseudoscalar diquarks are not allowed to exist as stable particles, but the scalar diquarks might be stable only at a rather strong interaction in the diquark channel. In the case of the CFL phase, all NG bosons of the model are realized as scalar and pseudoscalar diquarks. Moreover, it turns out that massive diquark excitations are unstable for this phase. In particular, for the scalar and pseudoscalar octets of diquark resonances a mass value around 230 MeV was found numerically. In contrast, mesons are stable particles in the CFL phase. Their masses lie in the interval $400\div500$ MeV for not too large values of $\mu > \mu_c$.

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I. INTRODUCTION

It is well-known that at asymptotically high baryon densities the ground state of massless three-flavor QCD corresponds to the so-called color – flavor locked (CFL) phase [1, 2]. In this phase quarks of all three flavors as well as three colors undergo pairing near the Fermi surface due to the attractive one-gluon exchange potential. The properties of different collective modes, including Nambu – Goldstone (NG) bosons, of the CFL phase were studied already in the framework of weak-coupling QCD [3]. At intermediate baryon densities, related to compact star physics, weak-coupling expansion of QCD is not applicable, so the description of color superconductivity, including the CFL phase, is more adequate in the framework of effective theories for the low energy QCD region. In particular, since massless excitations might play an important role in different transport phenomena, such as cooling processes of neutron stars etc, different chiral type effective theories for the pseudoscalar NG bosons of the CFL phase are usually used (see, e.g., [4, 5]).

Another effective theory approach is based on the Nambu – Jona-Lasinio (NJL) models. Since any NJL model contains the microscopic quark degrees of freedom, it is especially convenient for the investigation of dynamical processes in dense baryonic matter. In particular, in the three-flavor NJL model the CFL effect was already considered, e.g., in [6], (see also the review [7]), where some aspects of the phase structure of dense quark matter were discussed, including the influence of the s-quark bare mass, color- and electric charge neutrality conditions, external magnetic field, etc. In addition, in [8, 9] the properties and structure of NG bosons of the CFL phase were considered in the framework of NJL models.

One of the most noticeable differences between color superconductivity phenomena with three and two quark species is that the CFL effect is characterized by a hierarchy of energy scales. As it was established in different approaches quoted above, at the lowest scale lie NG bosons, which dominate in all physical processes with energy smaller, than the superconducting gap Δ. Evident contributors at higher energy scales are quark quasiparticles, which in the CFL phase have an energy greater than Δ. However, up to now we know much less about other excitations, whose energy and mass are of the order of Δ in magnitude. Among these particles are ordinary scalar and pseudoscalar mesons, massive diquarks etc, i.e. particles which might play an essencial role in dynamical processes of the CFL phase. In contrast, the properties of mesons and diquarks, surrounded by color superconducting quark matter, were already discussed in the framework of the two-flavor NJL model [10, 11, 12, 13, 14].

In the present paper we are going to study just this type of excitations of the CFL ground state, i.e. mesons and massive diquarks, in the framework of the massless three-flavor NJL model. In our previous paper [9] we have, in particular, obtained the equations for both scalar and pseudoscalar diquark masses in the CFL phase of the NJL model. There, our consideration was based on the effective action, which is a generating functional for one-particle irreducible Green functions. Now, using the same technique, we perform a numerical investigation of diquark masses, as well as of the masses of scalar and pseudoscalar mesons, vs the chemical potential in the CFL phase. Moreover, the octet and singlet structure is established for massive mesons and diquarks (both scalar and pseudoscalar) in the CFL phase. In addition, the masses of diquarks and mesons in the chirally broken quark matter phase are also investigated.

II. NJL MODEL AND ITS EFFECTIVE ACTION

Let us consider the following NJL model with three massless quark flavors

$$L = \bar{q}\Big[\gamma^\nu i\partial_\nu + \mu\gamma^0\Big]q + G_1\sum_{a=0}^{8}\Big[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma^5\tau_a q)^2\Big] + \\ + \; G_2\sum_{A=2,5,7}\;\sum_{A'=2,5,7}\Big\{[\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q][\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C] + [\bar{q}^C\tau_A\lambda_{A'}q][\bar{q}\tau_A\lambda_{A'}q^C]\Big\}. \tag{1}$$

In (1), $\mu \geq 0$ is the quark number chemical potential which is the same for all quark flavors, $q^C = C\bar{q}^t$, $\bar{q}^C = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). The quark field $q \equiv q_{i\alpha}$ is a flavor and color triplet as well as a four-component Dirac spinor, where $i, \alpha = 1, 2, 3$. (Roman and Greek indices refer to flavor and color indices, respectively; spinor indices are omitted.) Furthermore, we use the notations τ_a, λ_a for Gell-Mann matrices in the flavor and color space, respectively ($a = 1, ..., 8$); $\tau_0 = \sqrt{\frac{2}{3}}\cdot\mathbf{1_f}$ is proportional to the unit matrix in the flavor space. Clearly, the Lagrangian (1) as a whole is invariant under transformations from the color group SU$(3)_c$. In addition, it is symmetric under the chiral group SU$(3)_L\times$SU$(3)_R$ (chiral transformations act on the flavor indices of quark fields only) as well as under the baryon-number conservation group U$(1)_B$ and the axial group U$(1)_A$. [1] In all numerical calculations below, we use the following values of the model parameters (see, e.g., ref. [7]): $\Lambda = 602.3$ MeV, $G_1\Lambda^2 = 2.319$ and $G_2 = 3G_1/4$, where Λ is an ultraviolet cutoff parameter in the three-dimensional momentum space.

The linearized version of the Lagrangian (1) contains collective bosonic fields $\sigma_a(x), \pi_a(x), \Delta^s_{AA'}(x), \Delta^p_{AA'}(x)$ and looks like

$$\begin{aligned}\tilde{L} \;=\; & \bar{q}\Big[\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma_a\tau_a - i\gamma^5\pi_a\tau_a\Big]q - \frac{1}{4G_1}\Big[\sigma_a\sigma_a + \pi_a\pi_a\Big] - \frac{1}{4G_2}\Big[\Delta^{s*}_{AA'}\Delta^s_{AA'} + \Delta^{p*}_{AA'}\Delta^p_{AA'}\Big] \\ - \; & \frac{\Delta^{s*}_{AA'}}{2}[\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q] - \frac{\Delta^s_{AA'}}{2}[\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C] - \frac{\Delta^{p*}_{AA'}}{2}[\bar{q}^C\tau_A\lambda_{A'}q] - \frac{\Delta^p_{AA'}}{2}[\bar{q}\tau_A\lambda_{A'}q^C],\end{aligned} \tag{2}$$

where here and in the following the summation over repeated indices $a = 0, ..., 8$ and $A, A' = 2, 5, 7$ is implied. Lagrangians (1) and (2) are equivalent which simply follows from the equations of motion for the bosonic fields

$$\begin{aligned}&\sigma_a(x) = -2G_1(\bar{q}\tau_a q), \quad \Delta^s_{AA'}(x) = -2G_2(\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q), \quad \Delta^{s*}_{AA'}(x) = -2G_2(\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C),\\ &\pi_a(x) = -2G_1(\bar{q}i\gamma^5\tau_a q), \quad \Delta^p_{AA'}(x) = -2G_2(\bar{q}^C\tau_A\lambda_{A'}q), \quad \Delta^{p*}_{AA'}(x) = -2G_2(\bar{q}\tau_A\lambda_{A'}q^C).\end{aligned} \tag{3}$$

In (2)-(3) $\sigma_a(x), \Delta^s_{AA'}(x)$ and $\pi_a(x), \Delta^p_{AA'}(x)$ are scalar and pseudoscalar fields, correspondingly.

Let us consider the flavor group SU$(3)_f$=SU$(3)_{L+R}$, which is the diagonal subgroup of the chiral group. Then, all complex scalar diquark fields $\Delta^s_{AA'}(x)$ form an $(\bar{3}_c, \bar{3}_f)$ multiplet of the SU$(3)_c\times$SU$(3)_f$ group, i.e. they are a color and flavor antitriplet. The same is true for complex pseudoscalar diquark fields $\Delta^p_{AA'}(x)$ which are also the components of an $(\bar{3}_c, \bar{3}_f)$-multiplet of the SU$(3)_c\times$SU$(3)_f$ group. Evidently, all diquarks $\Delta^{s,p}_{AA'}(x)$ have the same nonzero baryon charge. All the real $\sigma_a(x)$ and $\pi_a(x)$ fields are color singlets. Moreover, the set of scalar $\sigma_a(x)$ mesons is decomposed into a direct sum of the singlet and octet representations of the diagonal flavor group SU$(3)_f$. The same decomposition into multiplets is true for the set of all pseudoscalar $\pi_a(x)$ mesons. Clearly, in this case the octet is constructed from three pions ($\pi^\pm$ and π^0), four kaons (K^0, $\bar{K}^0$ and $K^\pm$) and the eta-meson (η_8), whereas the singlet (η_0) corresponds to the η' meson.

In our previous paper [9], using the intermediate bosonic Lagrangian (2) and the Nambu–Gorkov formalism we have obtained in the one-fermion loop approximation the effective action $\mathcal{S}_{\rm eff}$ of the initial model (1). In terms of collective bosonic fields (3) it takes the following form:

$$\mathcal{S}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = -\int d^4x\left[\frac{\sigma_a^2 + \pi_a^2}{4G_1} + \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2}\right] - \frac{i}{2}{\rm Tr}_{scfxNG}\ln Z, \tag{4}$$

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[1] In a more realistic case, the additional ‘t Hooft six-quark interaction term should be taken into account in order to break the axial U$(1)_A$ symmetry [6]. However, in the present consideration we omit the ‘t Hooft term, for simplicity.

where Z is the 2×2-matrix in the Nambu–Gorkov space,

$$Z = \begin{pmatrix} D^+, & -K \\ -K^*, & D^- \end{pmatrix}, \tag{5}$$

and the following notations are adopted

$$\begin{array}{lll} D^+ = i\gamma^\nu\partial_\nu + \mu\gamma^0 - \Sigma, & \Sigma = \tau_a\sigma_a + i\gamma^5\pi_a\tau_a, & K = (\Delta^p_{AA'} + i\Delta^s_{AA'}\gamma^5)\tau_A\lambda_{A'}, \\ D^- = i\gamma^\nu\partial_\nu - \mu\gamma^0 - \Sigma^t, & \Sigma^t = \tau^t_a\sigma_a + i\gamma^5\pi_a\tau^t_a, & K^* = (\Delta^{p*}_{AA'} + i\Delta^{s*}_{AA'}\gamma^5)\tau_A\lambda_{A'}. \end{array} \tag{6}$$

Besides of an evident trace over the two-dimensional Nambu–Gorkov (NG) matrix, the Tr-operation in (4) stands for the trace in spinor (s), flavor (f), color (c) as well as four-dimensional coordinate (x) spaces, correspondingly. Let us suppose that parity is conserved so that all pseudoscalar diquark and meson fields have zero ground state expectation values, i.e. $\langle\Delta^p_{AA'}(x)\rangle = 0$ and $\langle\pi_a(x)\rangle = 0$. Furthermore, since at zero s-quark mass, $m_s = 0$, only the competition between the chirally broken quark matter phase and the CFL one is relevant to the physics of dense QCD (see, e.g., [2]), we permit in the present consideration nonzero ground state expectation values only for $\sigma_0(x)$ and $\Delta^s_{AA}(x)$ fields ($A = 2,5,7$). Namely, let $\langle\sigma_0(x)\rangle \equiv \sigma$, $\langle\Delta^s_{AA}(x)\rangle \equiv \Delta$, $\langle\Delta^{s*}_{AA}(x)\rangle \equiv \Delta^*$, where $A = 2,5,7$, but other boson fields from (3) have zero ground state expectation values. In the case $\Delta = 0$, $\sigma \neq 0$ quark matter is in the chirally broken phase, where the ground state is invariant under SU(3)$_c\times$SU(3)$_f\times$U(1)$_B$. If $\Delta \neq 0$, then the CFL phase is realized in the model, and the initial symmetry is spontaneously broken down to SU(3)$_{L+R+c}$. [2] Now, let us make the following shifts of bosonic fields in (4): $\sigma_0(x) \to \sigma_0(x) + \sigma$, $\Delta^{s*}_{AA}(x) \to \Delta^{s*}_{AA}(x) + \Delta^*$, $\Delta^s_{AA}(x) \to \Delta^s_{AA}(x) + \Delta$, $(A = 2,5,7)$, and other bosonic fields remain unshifted. (Obviously, the new shifted bosonic fields $\sigma_0(x), \Delta^s_{AA}(x)$ etc, now denote the (small) quantum fluctuations around the mean values σ, Δ etc of mesons and diquarks rather than the original fields (3).) In this case

$$Z \to \begin{pmatrix} D^+_o\,, & -K_o \\ -K^*_o\,, & D^-_o \end{pmatrix} - \begin{pmatrix} \Sigma\,, & K \\ K^*\,, & \Sigma^t \end{pmatrix} \equiv S_0^{-1} - \begin{pmatrix} \Sigma\,, & K \\ K^*\,, & \Sigma^t \end{pmatrix}, \tag{7}$$

where $K_o, K^*_o, D^\pm_o, \Sigma_o, \Sigma^t_o$ are the corresponding quantities (6), in which all bosonic fields are replaced by their own ground state expectation values, i.e. $\sigma_0(x) \to \sigma$, $\pi_a(x) \to 0$, $\Delta^s_{AA}(x) \to \Delta$, $\Delta^p_{AA'}(x) \to 0$ etc, and S_0 is the quark propagator matrix in the Nambu–Gorkov representation (its matrix elements S_{ij} are given in Appendix B). Then, expanding the obtained expression into a Taylor-series up to second order of small bosonic fluctuations, we have

$$\mathcal{S}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = \mathcal{S}^{(0)}_{\rm eff} + \mathcal{S}^{(2)}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) + \cdots, \tag{8}$$

where

$$\mathcal{S}^{(0)}_{\rm eff} = -\int d^4x \left[\frac{\sigma\sigma}{4G_1} + \frac{3|\Delta|^2}{4G_2}\right] + \frac{i}{2}{\rm Tr}_{scfxNG}\ln(S_0) \equiv -\Omega(\sigma,\Delta,\Delta^*)\int d^4x, \tag{9}$$

$$\begin{aligned} \mathcal{S}^{(2)}_{\rm eff}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = &-\int d^4x \left[\frac{\sigma_a^2 + \pi_a^2}{4G_1} + \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2}\right] + \\ &+ \frac{i}{4}{\rm Tr}_{scfxNG}\left\{S_0 \begin{pmatrix} \Sigma\,, & K \\ K^*\,, & \Sigma^t \end{pmatrix} S_0 \begin{pmatrix} \Sigma\,, & K \\ K^*\,, & \Sigma^t \end{pmatrix}\right\}, \end{aligned} \tag{10}$$

and $\Omega(\sigma,\Delta,\Delta^*)$ is the thermodynamic potential of the system. Notice that the term linear in meson and diquark fields vanishes in (8) due to the gap equations.

The detailed investigations of the thermodynamic potential, performed in our previous paper [9] for the above accepted model parameter set, shows that at $\mu < \mu_c \approx 330$ MeV the chirally broken quark matter phase with SU(3)$_c\times$SU(3)$_f\times$U(1)$_B$ symmetric ground state is realized in the model (in this case $\Delta = 0$ and $M \approx 355$ MeV, where $M = \sqrt{2/3}\,\sigma$ is the dynamical quark mass). However, at $\mu > \mu_c$ the CFL phase of dense baryonic matter arises. In this phase $M = 0$ and Δ varies with μ (see Fig. 2 in [9]). Below we suppose that the gap Δ is a real nonnegative number.

In the following we will study the spectrum of meson/diquark excitations both in the CFL and chirally broken phases of the NJL model. Since particle masses are calculated by the use of corresponding Green functions, it is necessary to put a special attention to the effective action $\mathcal{S}^{(2)}_{\rm eff}$ (10) which is really a generating functional of the

[2] In spite of the fact that in the CFL phase the chiral symmetry is also broken, the notation "chirally broken phase" is used here and in the following for the phase without color superconductivity.

one-particle irreducible (1PI) two-point Green functions of mesons and diquarks both in the chirally broken and CFL phases, namely

$$\Gamma_{XY}(x-y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\rm eff}}{\delta Y(y)\delta X(x)}, \tag{11}$$

where $X(x), Y(x) = \sigma_a(x), \pi_b(x), \Delta^{s,p}_{AA'}(x), \Delta^{s,p*}_{BB'}(x)$. (To obtain a Green function (11) in the chirally broken phase of quark matter ($\mu < \mu_c$), one should use in the expression for the quark propagator S_0 (see Appendix B), entering in (10), $M \approx 355$ MeV and $\Delta = 0$, whereas in the CFL phase ($\mu > \mu_c$) all Green functions (11) correspond to S_0 with $M = 0$ and values $\Delta \neq 0$ presented in Fig. 2 of [9].) In the following, we shall say that in the theory there is a *mixing* between two different particles with corresponding fields $X(x)$ and $Y(x)$, if their 1PI Green function $\Gamma_{XY}(x-y)$ is not identically equal to zero. Now, after performing in (10) the trace operation over the two-dimensional Nambu-Gorkov space, we obtain

$$\mathcal{S}^{(2)}_{\rm eff} = \mathcal{S}^{(2)}_{\rm mesons} + \mathcal{S}^{(2)}_{\rm diquarks} + \mathcal{S}^{(2)}_{\rm mixed}, \tag{12}$$

where

$$\mathcal{S}^{(2)}_{\rm mesons} = -\int d^4x \frac{\sigma_a^2 + \pi_a^2}{4G_1} + \frac{i}{4}{\rm Tr}_{scfx}\left\{S_{11}\Sigma S_{11}\Sigma + 2S_{12}\Sigma^t S_{21}\Sigma + S_{22}\Sigma^t S_{22}\Sigma^t\right\}, \tag{13}$$

$$\mathcal{S}^{(2)}_{\rm diquarks} = -\int d^4x \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2} + \frac{i}{4}{\rm Tr}_{scfx}\left\{S_{12}K^* S_{12}K^* + 2S_{11}KS_{22}K^* + S_{21}KS_{21}K\right\}, \tag{14}$$

$$\mathcal{S}^{(2)}_{\rm mixed} = \frac{i}{2}{\rm Tr}_{scfx}\left\{S_{11}\Sigma S_{12}K^* + S_{21}\Sigma S_{11}K + S_{12}\Sigma^t S_{22}K^* + S_{21}KS_{22}\Sigma^t\right\}, \tag{15}$$

and S_{ij} are the matrix elements of the quark propagator matrix S_0 defined in (7) (see also Appendix B). (Some necessary explanations concerning the trace-operation over coordinate space in the expressions (13)-(15) are given in Appendix A (see (A4))). It follows from these formulae that the effective action (13) depends on the mesonic fields $\sigma_a(x), \pi_b(x)$ only, i.e. it is a generating functional of the two-point 1PI Green functions of mesons. Furthermore, the effective action (14) is composed from diquark fields only, and the mixing between mesons and diquarks might occur because of the effective action (15). However, as a detailed analysis of the NJL model (1) with three massless quarks shows, each Green function, containing mixing of mesons and diquarks, is proportional to $M\Delta$. Thus, both in the chirally broken phase of quark matter, where $\Delta = 0$, and in the CFL one, where $M = 0$, there is no mixing between mesons and diquarks, and so the term (15) can safely be ignored in our present consideration. [3]

Finally, note that because of the traces containing an odd number of γ^5 matrices, there is no mixing between scalar and pseudoscalar particles in the framework of the NJL model (1), as required by parity conservation.

III. MESON MASSES

A. The case of the CFL phase

Taking into account the remarks from the previous section, we have the following general expressions for the nonzero two-point 1PI Green function of mesons which are valid in the chirally broken phase of quark matter as well as in the CFL one (other two-point mesonic 1PI Green functions are zero in the model under consideration):

$$\Gamma_{\sigma_a\sigma_a}(x-y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\rm mesons}}{\delta\sigma_a(y)\delta\sigma_a(x)}, \qquad \Gamma_{\pi_b\pi_b}(x-y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\rm mesons}}{\delta\pi_b(y)\delta\pi_b(x)} \tag{16}$$

where $a, b = 0, 1, 2, ..., 8$. In momentum space the zeros of the Fourier transformations of these functions are connected with meson masses.

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[3] Note, if some of the current quark masses are nonzero, then due to $M \neq 0$ there arises a mixing between mesons and diquarks in the CFL phase. This effect is analogous to the mixing between the σ-meson and the scalar diquark in the color superconducting phase of a two-flavor NJL model with nonzero masses of u- and d-quarks [11, 12].

Starting from (16), it is possible to obtain the expressions for the mesonic 1PI Green functions $(a, b = 0, 1, 2, ..., 8)$

$$\Gamma_{\pi_a\pi_a}(z) = \frac{\delta(z)}{2G_1} + \frac{i}{2}\mathrm{Tr}_{scf}\left[S_{11}(z)\gamma^5\tau_a S_{11}(-z)\gamma^5\tau_a + S_{12}(z)\gamma^5\tau_a^t S_{21}(-z)\gamma^5\tau_a\right.$$
$$\left. + S_{21}(z)\gamma^5\tau_a S_{12}(-z)\gamma^5\tau_a^t + S_{22}(z)\gamma^5\tau_a^t S_{22}(-z)\gamma^5\tau_a^t\right], \tag{17}$$

$$\Gamma_{\sigma_b\sigma_b}(z) = \frac{\delta(z)}{2G_1} - \frac{i}{2}\mathrm{Tr}_{scf}\left[S_{11}(z)\tau_b S_{11}(-z)\tau_b + S_{12}(z)\tau_b^t S_{21}(-z)\tau_b\right.$$
$$\left. + S_{21}(z)\tau_b S_{12}(-z)\tau_b^t + S_{22}(z)\tau_b^t S_{22}(-z)\tau_b^t\right]. \tag{18}$$

In (17)-(18) $z = x - y$ and the matrix elements $S_{ij}(z)$ are presented in formulae (B1)-(B4), from which the Fourier transformations $\overline{S}_{ij}(p)$ are directly seen. The corresponding Fourier transformations $\overline{\Gamma}_{\pi_a\pi_a}(p)$ and $\overline{\Gamma}_{\sigma_b\sigma_b}(p)$ look like (as an example, see the relation (A7) from Appendix A):

$$\overline{\Gamma}_{\pi_a\pi_a}(p) = \frac{1}{2G_1} + \frac{i}{2}\mathrm{Tr}_{scf}\int\frac{d^4q}{(2\pi)^4}\left[\,\overline{S}_{11}(p+q)\gamma^5\tau_a\overline{S}_{11}(q)\gamma^5\tau_a + \overline{S}_{12}(p+q)\gamma^5\tau_a^t\overline{S}_{21}(q)\gamma^5\tau_a\right.$$
$$\left. + \overline{S}_{21}(p+q)\gamma^5\tau_a\overline{S}_{12}(q)\gamma^5\tau_a^t + \overline{S}_{22}(p+q)\gamma^5\tau_a^t\overline{S}_{22}(q)\gamma^5\tau_a^t\right], \tag{19}$$

$$\overline{\Gamma}_{\sigma_b\sigma_b}(p) = \frac{1}{2G_1} - \frac{i}{2}\mathrm{Tr}_{scf}\int\frac{d^4q}{(2\pi)^4}\left[\,\overline{S}_{11}(p+q)\tau_b\overline{S}_{11}(q)\tau_b + \overline{S}_{12}(p+q)\tau_b^t\overline{S}_{21}(q)\tau_b\right.$$
$$\left. + \overline{S}_{21}(p+q)\tau_b\overline{S}_{12}(q)\tau_b^t + \overline{S}_{22}(p+q)\tau_b^t\overline{S}_{22}(q)\tau_b^t\right]. \tag{20}$$

The zeros of these functions determine the π- and σ-meson dispersion laws, i.e. the relations between their energy and three-momenta. In the present paper, we are mainly interested in the investigation of the modification of meson and diquark masses in dense and cold quark matter. Since in this case a particle mass is defined as the value of its energy in the rest frame, $\vec{p} = 0$ (see, e.g., [11, 12, 13, 15]), we put $p = (p_0, 0, 0, 0)$ in the following. As a result, the calculation of two-point 1PI Green functions is significantly simplified. In order to perform for the case of the CFL phase in (19)-(20) the trace operations over color and flavor spaces, we used the program of analytical calculations MAPLE. The trace over spinor space and the subsequent integration over q_0, has been performed by applying the technique elaborated in [11, 12]. As a result, we have for the 1PI meson Green functions with $a = 1, 2, ..., 8$:

$$\overline{\Gamma}_{\sigma_a\sigma_a}(p_0) = \frac{1}{2G_1} + \mathcal{A} - \mathcal{B}, \qquad \overline{\Gamma}_{\pi_a\pi_a}(p_0) = \frac{1}{2G_1} + \mathcal{A} + \mathcal{B}, \tag{21}$$

where

$$\mathcal{A} = \int\frac{d^3q}{(2\pi)^3}\left\{\frac{28(E_\Delta^+ + E_\Delta^-)[E_\Delta^+E_\Delta^- + E^+E^-]}{3E_\Delta^+E_\Delta^-[p_0^2 - (E_\Delta^+ + E_\Delta^-)^2]} + \frac{4(E_{2\Delta}^+ + E_\Delta^-)[E_{2\Delta}^+E_\Delta^- + E^+E^-]}{3E_{2\Delta}^+E_\Delta^-[p_0^2 - (E_{2\Delta}^+ + E_\Delta^-)^2]} +\right.$$
$$\left. + \frac{4(E_\Delta^+ + E_{2\Delta}^-)[E_\Delta^+E_{2\Delta}^- + E^+E^-]}{3E_\Delta^+E_{2\Delta}^-[p_0^2 - (E_\Delta^+ + E_{2\Delta}^-)^2]}\right\}, \tag{22}$$

$$\mathcal{B} = \int\frac{d^3q}{(2\pi)^3}\left\{\frac{8\Delta^2(E_\Delta^+ + E_\Delta^-)}{3E_\Delta^+E_\Delta^-[p_0^2 - (E_\Delta^+ + E_\Delta^-)^2]} + \frac{8\Delta^2(E_{2\Delta}^+ + E_\Delta^-)}{3E_{2\Delta}^+E_\Delta^-[p_0^2 - (E_{2\Delta}^+ + E_\Delta^-)^2]} +\right.$$
$$\left. + \frac{8\Delta^2(E_\Delta^+ + E_{2\Delta}^-)}{3E_\Delta^+E_{2\Delta}^-[p_0^2 - (E_\Delta^+ + E_{2\Delta}^-)^2]}\right\}. \tag{23}$$

Moreover, we use in these formulae the notations $(E_\Delta^\pm)^2 = (E^\pm)^2 + |\Delta|^2$, $(E_{2\Delta}^\pm)^2 = (E^\pm)^2 + 4|\Delta|^2$, $E^\pm = E \pm \mu$, $E = \sqrt{\vec{q}^2 + M^2}$, in which M is set equal to zero. It is clear from (22)-(23) that each of the Green functions (21) depends on p_0^2. So the mass squared of the a-th scalar (or pseudoscalar) meson ($a = 1, 2, ..., 8$) is determined by a zero of the function $\overline{\Gamma}_{\sigma_a\sigma_a}(p_0)$ (or $\overline{\Gamma}_{\pi_a\pi_a}(p_0)$) in the p_0^2 plane. Moreover, it is evident from (21) that all scalar mesons with $a = 1, 2, ..., 8$ have the same mass in the CFL phase, thus forming an SU(3) octet of scalar mesons. In a similar way, all pseudoscalar mesons with $a = 1, 2, ..., 8$ form another massive SU(3) octet as well. The masses of scalar and pseudoscalar meson octets in the CFL phase are presented in Fig. 1 and 2, correspondingly.

In contrast, the two-point Green functions for $\sigma_0(x)$ and $\pi_0(x)$ mesons take another form. Indeed,

$$\overline{\Gamma}_{\sigma_0\sigma_0}(p_0) = \frac{1}{2G_1} + \mathcal{Q} + \mathcal{R}, \qquad \overline{\Gamma}_{\pi_0\pi_0}(p_0) = \frac{1}{2G_1} + \mathcal{Q} - \mathcal{R}, \tag{24}$$

where

$$\mathcal{Q} = \int\frac{d^3q}{(2\pi)^3}\left\{\frac{32(E_\Delta^+ + E_\Delta^-)[E_\Delta^+E_\Delta^- + E^+E^-]}{3E_\Delta^+E_\Delta^-[p_0^2 - (E_\Delta^+ + E_\Delta^-)^2]} + \frac{4(E_{2\Delta}^+ + E_{2\Delta}^-)[E_{2\Delta}^+E_{2\Delta}^- + E^+E^-]}{3E_{2\Delta}^+E_{2\Delta}^-[p_0^2 - (E_{2\Delta}^+ + E_{2\Delta}^-)^2]}\right\}, \tag{25}$$

$$\mathcal{R} = \int\frac{d^3q}{(2\pi)^3}\left\{\frac{32\Delta^2(E_\Delta^+ + E_\Delta^-)}{3E_\Delta^+E_\Delta^-[p_0^2 - (E_\Delta^+ + E_\Delta^-)^2]} + \frac{16\Delta^2(E_{2\Delta}^+ + E_{2\Delta}^-)}{3E_{2\Delta}^+E_{2\Delta}^-[p_0^2 - (E_{2\Delta}^+ + E_{2\Delta}^-)^2]}\right\} \tag{26}$$

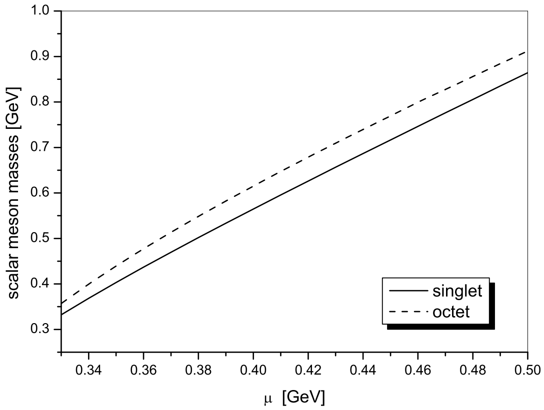

FIG. 1: The behaviour of the scalar meson masses vs μ in the CFL phase.

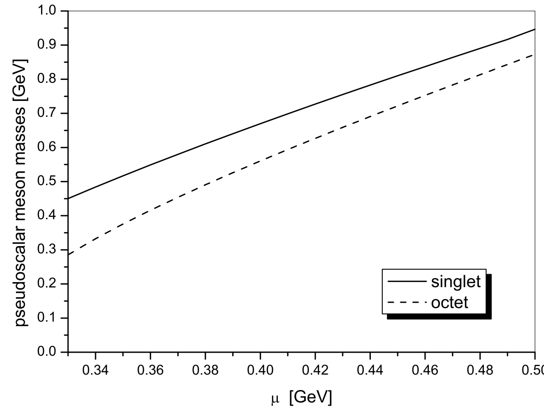

FIG. 2: The behaviour of the pseudoscalar meson masses vs μ in the CFL phase.

(in (25)-(26) the quantities $E^{\pm}$ etc are taken again at $M = 0$). Evidently, these mesons are singlets with respect to the SU(3) group, and their masses are presented also in Figs 1,2. It is clear from these figures that none of the mesons have a zero mass in the CFL phase, i.e. they are not the Nambu – Goldstone bosons (NG) of this phase. Moreover, one can see that in the CFL phase there is a singlet-octet mass splitting of pseudoscalar and scalar mesons, which however vanishes in the $\Delta = 0$ limit. Indeed, if the value $\Delta = 0$ is used in (21)-(26), then the 1PI Green functions of the octet and singlet mesons are the same, i.e. the mass splitting is absent.

B. The case of chirally broken quark matter phase

Now, let us find the meson masses in the chirally broken phase of quark matter, i.e. at at $\mu < \mu_c$, $M \approx 355$ MeV, and $\Delta = 0$. In this case the calculation of the traces over color and flavor indices in (19)-(20) is greatly simplified, so in the rest frame, $p = (p_0, 0, 0, 0)$, the mesonic Green functions look like $(a, b = 0, 1, 2, ..., 8)$:

$$\overline{\Gamma}_{\pi_a\pi_a}(p_0) = p_0^2 \int \frac{d^3q}{(2\pi)^3} \cdot \frac{12}{E[p_0^2 - 4E^2]}, \qquad \overline{\Gamma}_{\sigma_b\sigma_b}(p_0) = (p_0^2 - 4M^2) \int \frac{d^3q}{(2\pi)^3} \cdot \frac{12}{E[p_0^2 - 4E^2]}. \tag{27}$$

(In obtaining (27), the gap equation (see, e.g., [9]) was used in order to eliminate the coupling constant G_1 from the expressions (19)-(20). Note also, the expressions (27) do not follow directly from (21)-(26) in the $\Delta = 0$ limit.) Evidently, in the chirally broken phase the Green functions $\overline{\Gamma}_{\pi_a\pi_a}(p_0)$ turn into zero at $p_0^2 = 0$ for all $a = 0, 1, ..., 8$. It means that nine massless excitations, Nambu – Goldstone bosons, do exist in the pseudoscalar meson sector of the model in the chirally broken phase. This fact corresponds to a spontaneous symmetry breaking of the initial $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_c \times \mathrm{U}(1)_B \times \mathrm{U}(1)_A$ down to the subgroup $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f \times \mathrm{U}(1)_B$ in the chirally broken phase. Moreover, it is clear from (27) that in this phase there exists a nonet of scalar mesons with mass $\equiv 2M \sim 710$ MeV.

In summary, we have three main conclusions about mesons in the framework of the NJL model (1) with three massless quarks. Firstly, we see that nine pseudoscalar mesons are NG bosons only in the chirally broken phase of quark matter. In the CFL phase they cease to be NG bosons, since now they acquire finite masses (see Fig. 2). Secondly, the CFL breaking of the symmetry generates the octet-singlet mass splitting of mesons. Thirdly, in the CFL phase the mass splitting among scalar and pseudoscalar mesons occurs differently. Indeed, at $\mu > \mu_c$ the mass of the scalar octet mesons is larger than the mass of the scalar singlet meson (see Fig. 1), whereas for pseudoscalar mesons the situation is inverse (see Fig. 2).

IV. DIQUARK MASSES

A. The case of the CFL phase

As it follows from the discussion in section II, all nonzero two-point 1PI Green function of diquarks both in the chirally broken and CFL phases might be determined through the relation

$$\Gamma_{XY}(x - y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\text{diquarks}}}{\delta Y(y) \delta X(x)}, \tag{28}$$

where the effective action $\mathcal{S}^{(2)}_{\rm diquarks}$ is given in (14), and $X(x),Y(x) = \Delta^s_{AA'}(x),\Delta^{s*}_{BB'}(x)$ or $X(x),Y(x) = \Delta^p_{AA'}(x),\Delta^{p*}_{BB'}(x)$. The last restriction again means that scalar and pseudoscalar diquarks do not mix in accord with parity conservation.

As shown in our earlier paper [12] for the case of the two-flavor NJL model, any two-point Green function of pseudoscalar diquarks differs in the color superconducting phase from a corresponding Green function of scalar diquarks by a term which is proportional to M^2, where M is the dynamical quark mass in this phase. The same is true for the CFL phase of the NJL model (1). So one can conclude that in the CFL phase of the model (1), where $M=0$, each Green function of pseudoscalar diquarks is equal to the corresponding Green function of scalar diquarks, e.g., $\Gamma_{\Delta^p_{AA'}\Delta^{p*}_{BB'}}(x-y)=\Gamma_{\Delta^s_{AA'}\Delta^{s*}_{BB'}}(x-y)$, etc. Hence, to establish the spectrum of the diquark excitations of the CFL phase, it is enough to study the set of scalar diquarks (the mass spectrum of the pseudoscalar diquark excitations will be the same in the CFL phase).

Let us consider the two-point 1PI Green functions of the scalar diquarks. A more detailed analysis of the effective action (14) shows that in the CFL phase, i.e. at $\mu>\mu_c$, where $M=0$ and $\Delta\neq 0$, eighteen scalar diquarks (nine $\Delta^s_{AA'}(x)$ and nine $\Delta^{s*}_{AA'}(x)$ fields) may be divided into four sectors: $s(57,75)$, $s(25,52)$, $s(27,72)$ and $s(257)$. Each of the sectors $s(AA',A'A)$, where $A\neq A'$, is composed of $\Delta^s_{AA'}(x)$, $\Delta^{s*}_{AA'}(x)$, $\Delta^s_{A'A}(x)$, and $\Delta^{s*}_{A'A}(x)$ diquark fields, whereas the sector $s(257)$ is composed of six fields, $\Delta^{s*}_{22}(x)$, $\Delta^{s*}_{55}(x)$, $\Delta^{s*}_{77}(x)$, $\Delta^s_{22}(x)$, $\Delta^s_{55}(x)$, and $\Delta^s_{77}(x)$. It turns out that there is a mixing between diquarks entering in the same sector, whereas fields from different sectors are not mixed. (The analogous situation takes place for the set of pseudoscalar diquarks, which is divided into nonmixing sectors $p(57,75)$, $p(25,52)$, $p(27,72)$ and $p(257)$.)

1. *The case of $s(AA',A'A)$ sectors*

Let us first study the mass spectrum of the excitations, e.g., in the sector $s(57,75)$. The two-point 1PI Green functions of scalar diquarks from this sector can be obtained from (14) by the relation (28). In the rest frame, where $p=(p_0,0,0,0)$, the Fourier transforms of these 1PI Green functions form the following matrix [9]:

$$\overline{\Gamma}_{57,75}(p_0)=\begin{pmatrix}\overline{\Gamma}_{\Delta^s_{57}\Delta^s_{57}}(p_0) & \overline{\Gamma}_{\Delta^s_{57}\Delta^{s*}_{57}}(p_0) & \overline{\Gamma}_{\Delta^s_{57}\Delta^s_{75}}(p_0) & \overline{\Gamma}_{\Delta^s_{57}\Delta^{s*}_{75}}(p_0)\\ \overline{\Gamma}_{\Delta^{s*}_{57}\Delta^s_{57}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{57}\Delta^{s*}_{57}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{57}\Delta^s_{75}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{57}\Delta^{s*}_{75}}(p_0)\\ \overline{\Gamma}_{\Delta^s_{75}\Delta^s_{57}}(p_0) & \overline{\Gamma}_{\Delta^s_{75}\Delta^{s*}_{57}}(p_0) & \overline{\Gamma}_{\Delta^s_{75}\Delta^s_{75}}(p_0) & \overline{\Gamma}_{\Delta^s_{75}\Delta^{s*}_{75}}(p_0)\\ \overline{\Gamma}_{\Delta^{s*}_{75}\Delta^s_{57}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{75}\Delta^{s*}_{57}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{75}\Delta^s_{75}}(p_0) & \overline{\Gamma}_{\Delta^{s*}_{75}\Delta^{s*}_{75}}(p_0)\end{pmatrix}\equiv\begin{pmatrix}0 & A & C & 0\\ B & 0 & 0 & C\\ C & 0 & 0 & A\\ 0 & C & B & 0\end{pmatrix}, \tag{29}$$

where $A\equiv\alpha+p_0\beta$, $B\equiv\alpha-p_0\beta$ and

$$\alpha=\int\frac{d^3q}{(2\pi)^3}\left\{\frac{6E^+_\Delta p_0^2-(E^+_\Delta+E^+_{2\Delta})^2(2E^+_\Delta+E^+_{2\Delta})}{9E^+_\Delta E^+_{2\Delta}[p_0^2-(E^+_\Delta+E^+_{2\Delta})^2]}+\frac{4p_0^2+4(E^+_\Delta)^2-10\Delta^2}{3E^+_\Delta[p_0^2-4(E^+_\Delta)^2]}\right\}+ \\ +\int\frac{d^3q}{(2\pi)^3}\Big\{E^+_\Delta\to E^-_\Delta,\quad E^+_{2\Delta}\to E^-_{2\Delta}\Big\}, \tag{30}$$

$$\beta=\int\frac{d^3q}{(2\pi)^3}\left\{\frac{E^+(E^+_\Delta+E^+_{2\Delta})}{3E^+_\Delta E^+_{2\Delta}[p_0^2-(E^+_\Delta+E^+_{2\Delta})^2]}+\frac{10E^+}{3E^+_\Delta[p_0^2-4(E^+_\Delta)^2]}\right\}- \\ -\int\frac{d^3q}{(2\pi)^3}\Big\{E^+\to E^-,\quad E^+_\Delta\to E^-_\Delta,\quad E^+_{2\Delta}\to E^-_{2\Delta}\Big\}, \tag{31}$$

$$C=\int\frac{d^3q}{(2\pi)^3}\left\{\frac{2\Delta^2(E^+_\Delta+E^+_{2\Delta})}{3E^+_\Delta E^+_{2\Delta}[p_0^2-(E^+_\Delta+E^+_{2\Delta})^2]}-\frac{10\Delta^2}{3E^+_\Delta[p_0^2-4(E^+_\Delta)^2]}\right\}+ \\ +\int\frac{d^3q}{(2\pi)^3}\Big\{E^+_\Delta\to E^-_\Delta,\quad E^+_{2\Delta}\to E^-_{2\Delta}\Big\}. \tag{32}$$

(To obtain the above expressions for α and β, one has to use the gap equation for Δ (see in [9]) in order to eliminate the coupling constant G_2 from corresponding 1PI Green functions.) Evidently, in the case of mixing between particles the information about their masses should be extracted from the zeros of the determinant of the matrix, composed from corresponding 1PI Green functions. So, in our case it is necessary to study the equation $\det\overline{\Gamma}_{57,75}(p_0)=0$, which takes the following form

$$\det\overline{\Gamma}_{57,75}(p_0)\equiv(AB-C^2)^2=[(\alpha-C)(\alpha+C)-p_0^2\beta^2]^2=0. \tag{33}$$

In the p_0^2-plane, each zero of this equation defines a mass squared of a bosonic excitation of the CFL phase ground state in the $s(57,75)$ sector. Since this sector contains four scalar diquarks, one should search for four solutions of the equation (33) in the p_0^2-plane. Clearly, due to the structure of (33), this equation admits at least two different solutions (each being two-fold degenerate), which are given by the zeros of the expression in the square bracket. It

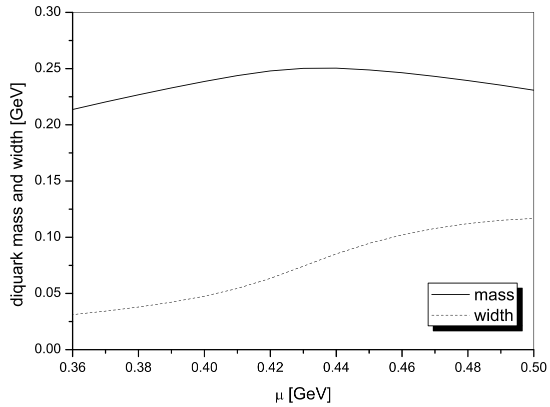

FIG. 3: The behaviour of the mass and the width of the scalar and pseudoscalar diquark octets vs μ in the CFL phase.

was proved in [9] that $\alpha - C \sim p_0^2$. Hence, the square bracket in (33) becomes zero at the point $p_0^2 = 0$. So, in the $s(57, 75)$ sector there are two massless scalar excitations, i.e. NG bosons.

Note, the expression $\det\overline{\Gamma}_{57,75}$ which is in the left hand side of the equation (33) is a complex-valued function defined on the Riemann manifold composed of an infinitely large number of sheets of the variable p_0^2. The first (physical) sheet is the p_0^2 plane with the cut $4\Delta^2 < p_0^2$ along the real axis. (Just the integrals (30)-(32) supply us with the values of the function $\det\overline{\Gamma}_{57,75}$ on this physical sheet.) It turns out that apart from the trivial zero, $p_0^2 = 0$, there are no solutions of the equation (33) on this sheet, so there are no stable massive diquark excitations in the $s(57, 75)$ sector. Using the procedure of analytical continuation presented in [11] we could find for each value of the chemical potential μ a complex point, lying on the second sheet of p_0^2, where the function $\det\overline{\Gamma}_{57,75}$ turns into zero. Evidently, the real and imaginary parts of this point correspond to the mass and width of a resonance. Hence, as was pointed out above, in the $s(57, 75)$ sector there appears a twicely degenerated excitation of the CFL phase, whose mass and width are presented in Fig. 3 as functions of μ.

A similar situation occurs in the other four-component sectors $s(25, 52)$ and $s(27, 72)$. Namely, for both sectors the 1PI Green function matrix has the form (29). Hence, in each of these sectors there are two NG bosons as well as two resonances with the same mass and width, given in Fig. 3.

2. Other diquark excitations of the CFL phase

First of all note that there are 36 (18 scalar- and 18 pseudoscalar-) diquark fields (3) in our model. So, there should exist at least 36 elementary diquark excitations both in the chirally broken quark matter phase and in the CFL one. The masses of 12 (six of them are NG bosons, the other six are massive resonances) scalar diquark excitations of the CFL phase were obtained in the previous section.

As it is clear from the discussion made at the beginning of section IV A, the mass spectrum of another 12 particles, which are the CFL ground state excitations in the pseudoscalar diquark sectors $p(57, 75)$, $p(25, 52)$, and $p(27, 72)$, is identical to the mass spectrum of the corresponding scalar diquarks from sectors $s(AA', A'A)$, where $A \neq A'$ (see the previous section IV A 1). Hence, in addition to the scalar excitations found in the previous section, in the CFL phase there exist six pseudoscalar NG bosons as well as six pseudoscalar resonances, whose mass and width are presented in Fig. 3.

Concerning the CFL ground state diquark excitations from the sectors $s(257)$ and $p(257)$, we should note that the corresponding 1PI Green functions form in the rest frame of the momentum representation a nontrivial 6×6 matrix $\overline{\Gamma}_{257}(p_0)$, which is the same both for the $s(257)$ and $p(257)$ diquark sectors, with a rather complicated determinant,

$$\det\overline{\Gamma}_{257}(p_0) = \big[(P - W)(Q - Z) - (R - T)^2\big]^2 \big\{(2T + R)^2 - (2W + P)(2Z + Q)\big\}. \tag{34}$$

An exact expression for the matrix $\overline{\Gamma}_{257}(p_0)$ as well as for the 1PI Green functions $P, Q, R, ...$ from (34) are presented in our previous paper [9], where it was also shown that the equation $\det\overline{\Gamma}_{257}(p_0) = 0$ has a three-fold degenerated solution $p_0^2 = 0$. So in the diquark sectors $s(257)$ and $p(257)$ there are six (three scalar and three pseudoscalar) NG excitations, and the initial NJL model (1) as a whole has 18 NG bosons in the mass spectrum of the CFL phase.

A more detailed consideration of the quantities $P, Q, R, ...$ shows that $P - W = A$, $Q - Z = B$, and $R - T = C$, where A, B, C are given in (29). So, the square brackets in (34) are no more than $\det\overline{\Gamma}_{57,75}(p_0)$ presented in (33). [4] As a consequence, we see, e.g., that in the $s(257)$ sector of the model there exist two massive resonances with the same mass and width, depicted in Fig. 3. Moreover, their mass and width are identical to those for the massive resonances from all scalar diquark sectors $s(AA', A'A)$. So, all these scalar diquark resonances form in total an octet with respect to the $\mathrm{SU}(3)_{L+R+c}$ group. A similar situation is valid for the pseudoscalar diquarks, where in the mass spectrum there is an octet of the CFL phase excitations with the same mass and width (see Fig. 3).

Unfortunately, we did not manage to find nontrivial diquark excitations of the CFL phase from the sectors $s(257)$ and $p(257)$, corresponding to a zero of the expression $F(p_0^2) \equiv (2T+R)^2 - (2W+P)(2Z+Q)$ that appears in the braces of (34). (Evidently, both excitations are $\mathrm{SU}(3)_{L+R+c}$ singlets.) The matter is that $F(p_0^2)$ is an analytical function on a rather complicated Riemann manifold of the variable p_0^2. On its first Riemann sheet there is only a trivial zero, $p_0^2 = 0$ (which corresponds to the NG bosons, as it was discussed above). Due to a rather complicated structure of the function $F(p_0^2)$, we were not able to perform its continuation onto the second Riemann sheet and get any information about the mass and width of the remaining $\mathrm{SU}(3)_{L+R+c}$ singlet diquark resonances from the sectors $s(257)$ and $p(257)$.

B. The case of the chirally broken phase of quark matter

In this phase, i.e. at $\mu < \mu_c$, the gap Δ vanishes, so the matrix elements $S_{12}(x-y)$ and $S_{21}(x-y)$ (see (B2) and (B3), correspondingly) of the quark propagator matrix S_0 are vanishing, too. As a consequence, the expression $\mathcal{S}^{(2)}_{\rm diquarks}$ of the two-point 1PI Green functions for diquark fields is simplified in the chirally broken phase:

$$\mathcal{S}^{(2)}_{\rm diquarks} = -\int d^4x \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2} + \frac{i}{2}\mathrm{Tr}_{scfx}\left\{S_{11}KS_{22}K^*\right\}. \tag{35}$$

Using this expression in (28), it is possible to get for each fixed $A, A' = 2, 5, 7$:

$$\Gamma_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(x-y) = \frac{\delta(x-y)}{4G_2} + \frac{i}{2}\mathrm{Tr}_{scf}\left\{S_{11}(x-y)\gamma^5\tau_A\lambda_{A'}S_{22}(y-x)\gamma^5\tau_A\lambda_{A'}\right\} \tag{36}$$

$$\Gamma_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(x-y) = \frac{\delta(x-y)}{4G_2} - \frac{i}{2}\mathrm{Tr}_{scf}\left\{S_{11}(x-y)\tau_A\lambda_{A'}S_{22}(y-x)\tau_A\lambda_{A'}\right\}. \tag{37}$$

In addition, the following relations are valid:

$$\Gamma_{\Delta^s_{AA'}\Delta^{s*}_{AA'}}(x-y) = \Gamma_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(y-x), \qquad \Gamma_{\Delta^p_{AA'}\Delta^{p*}_{AA'}}(x-y) = \Gamma_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(y-x), \tag{38}$$

and other two-point diquark 1PI Green functions are identically equal to zero in the chirally broken phase. Using the expressions (B5)-(B6) for the fermion Green functions, one can easily perform the Tr-operation over color and flavor indices in (36)-(37). Then, in the rest frame of the momentum representation, i.e. at $p = (p_0, 0, 0, 0)$, we have for each fixed pair of $A, A' = 2, 5, 7$:

$$\overline{\Gamma}_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(p_0) = \frac{1}{4G_2} - 16\int \frac{d^3q}{(2\pi)^3}\frac{E}{4E^2 - (p_0+2\mu)^2} \equiv \frac{1}{4G_2} - \Phi_s(\epsilon), \tag{39}$$

$$\overline{\Gamma}_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(p_0) = \frac{1}{4G_2} - 16\int \frac{d^3q}{(2\pi)^3}\frac{\vec{q}^2}{E}\frac{1}{4E^2 - (p_0+2\mu)^2} \equiv \frac{1}{4G_2} - \Phi_p(\epsilon), \tag{40}$$

where $\epsilon = (p_0+2\mu)^2$. Moreover, it follows from (38) that $\overline{\Gamma}_{\Delta^s_{AA'}\Delta^{s*}_{AA'}}(p_0) = \overline{\Gamma}_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(-p_0)$, $\overline{\Gamma}_{\Delta^p_{AA'}\Delta^{p*}_{AA'}}(p_0) = \overline{\Gamma}_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(-p_0)$. From the above general consideration of the diquark Green functions in the chirally broken quark matter phase we see that: i) scalar diquarks do not mix with pseudoscalar ones, ii) each scalar $\Delta^s_{AA'}$ or pseudoscalar $\Delta^p_{AA'}$ diquark field is mixed only with its complex conjugated one. Hence, for each pair of scalar diquarks $\Delta^s_{AA'}, \Delta^{s*}_{AA'}$ (or pseudoscalar diquarks $\Delta^p_{AA'}, \Delta^{p*}_{AA'}$, correspondingly) we have a simple 2×2 matrix $\overline{\Gamma}^s_{AA'}(p_0)$ of their 1PI Green functions (it is a 2×2 matrix $\overline{\Gamma}^p_{AA'}(p_0)$ for the system of two pseudoscalar diquarks $\Delta^p_{AA'}, \Delta^{p*}_{AA'}$, correspondingly):

$$\overline{\Gamma}^s_{AA'}(p_0) = \begin{pmatrix} 0 & \overline{\Gamma}_{\Delta^s_{AA'}\Delta^{s*}_{AA'}}(p_0) \\ \overline{\Gamma}_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(p_0) & 0 \end{pmatrix}, \quad \overline{\Gamma}^p_{AA'}(p_0) = \begin{pmatrix} 0 & \overline{\Gamma}_{\Delta^p_{AA'}\Delta^{p*}_{AA'}}(p_0) \\ \overline{\Gamma}_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(p_0) & 0 \end{pmatrix}. \tag{41}$$

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[4] This fact was not observed in [9], leading to an incorrect statement about the multiplet structure of the massive diquark excitations of the CFL phase.

Actually, due to the relations (39)-(40), these matrices do not depend on $A, A' = 2, 5, 7$, i.e. they are the same for each pair of scalar $\Delta^s_{AA'}, \Delta^{s*}_{AA'}$ or pseudoscalar $\Delta^p_{AA'}, \Delta^{p*}_{AA'}$ diquarks. Obviously, to obtain the diquark excitations of the chirally broken phase, it is sufficient to solve the equations $\det\overline{\Gamma}^s_{AA'}(p_0) = 0$ and $\det\overline{\Gamma}^p_{AA'}(p_0) = 0$ or the following ones

$$\overline{\Gamma}_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(p_0) \equiv \frac{1}{4G_2} - \Phi_s(\epsilon) = 0, \tag{42}$$

$$\overline{\Gamma}_{\Delta^{p*}_{AA'}\Delta^p_{AA'}}(p_0) \equiv \frac{1}{4G_2} - \Phi_p(\epsilon) = 0. \tag{43}$$

In the present consideration we restrict ourselves to looking only for stable diquark excitations of the chirally broken phase.

Note, the functions $\Phi_s(\epsilon)$ and $\Phi_p(\epsilon)$ are analytical in the whole complex ϵ-plane, except for the cut $4M^2 < \epsilon$ along the real axis. (In general, these functions are defined on complex Riemann surfaces which are to be described by several sheets. The integral representations for $\Phi_{s,p}(\epsilon)$, given in (39)-(40), define its values on the first sheet only. To find values of $\Phi_{s,p}(\epsilon)$ on the rest of the Riemann surfaces, a special procedure of analytical continuation is needed (see, e.g., in [11]).) Let us denote by ϵ^s_0 and ϵ^p_0 the solutions of the equations (42) and (43), respectively. Of course, they depend on the coupling constant G_2 of the diquark channel. Obviously, the stable diquark excitation corresponds to the root ϵ^s_0 which lies on the first Riemann sheet and obeys the constraint $0 < \epsilon^s_0 < 4M^2$. It is fulfilled only if $H^* < G_2 < H^{**}$, where H^* and H^{**} are defined by

$$\begin{aligned} H^* &\equiv \frac{1}{4\Phi_s(4M^2)} = \frac{\pi^2}{4\left[\Lambda\sqrt{M^2+\Lambda^2} + M^2\ln((\Lambda+\sqrt{M^2+\Lambda^2})/M)\right]}, \\ H^{**} &\equiv \frac{1}{4\Phi_s(0)} = \frac{\pi^2}{4\left[\Lambda\sqrt{M^2+\Lambda^2} - M^2\ln((\Lambda+\sqrt{M^2+\Lambda^2})/M)\right]} = \frac{3G_1}{2}. \end{aligned} \tag{44}$$

(Actually, the last equality in (44), i.e. $H^{**} = 3G_1/2$, is due to the gap equation for $M \neq 0$.) In this case ϵ^s_0 is the mass squared of the stable scalar diquark in the vacuum, i.e. at $\mu = 0$. For a rather weak interaction in the diquark channel ($G_2 < H^*$), ϵ^s_0 runs onto the second Riemann sheet, and unstable scalar diquark modes (resonances) appear. Unlike this, a sufficiently strong interaction in the diquark channel ($H^{**} < G_2$) pushes ϵ^s_0 towards the negative semi-axis of the first Riemann sheet, i.e. in this case $\epsilon^s_0 \equiv (M^o_D)^2 < 0$, where M^o_D is the mass of the diquark in the vacuum. The latter indicates a tachyon singularity in the scalar diquark propagator, evidencing that the $SU(3)_{L+R}\times SU(3)_c\times U(1)_B$ symmetric ground state of the chirally broken phase is not stable (in this case there arises a deeper ground state, corresponding to another phase of the model, the CFL phase). A similar observation was made in the framework of a two-flavor NJL model, where the chirally broken quark matter phase is unstable if there is a sufficiently strong interaction in the diquark channel [11, 12, 16]. Indeed, at a very large G_2, as it has been shown in [17], the color symmetry is spontaneously broken even at a vanishing chemical potential.

Let us ignore for a moment the scalar diquark sector and perform a similar analysis, based on the equation (43), for pseudoscalar diquark excitations. Then, pseudoscalar diquarks are stable excitations of the chirally broken phase only, if the constraint $H^{**} < G_2 < H^{***}$ is fulfilled, where H^{**} is given in (44) and

$$H^{***} = \frac{1}{4\Phi_p(0)} = \frac{\pi^2\Lambda\sqrt{M^2+\Lambda^2}}{4\left[3M^2\Lambda^2 + \Lambda^4 - 3M^2\Lambda\sqrt{M^2+\Lambda^2}\ln((\Lambda+\sqrt{M^2+\Lambda^2})/M)\right]}. \tag{45}$$

(In this case the solution ϵ^p_0 of the equation (43) lies inside the interval $0 < \epsilon^p_0 < 4M^2$.) It turns out that at $G_2 < H^{**}$ these excitations are unstable, whereas at $H^{***} < G_2$ a tachyonic instability of the chirally broken phase appears.

Now, combining together the above separate considerations of the scalar and pseudoscalar diquark excitations, we may conclude that at a rather weak interaction in the diquark channel ($G_2 < H^*$) both scalar and pseudoscalar diquark excitations of the chirally broken phase are resonances. If $H^* < G_2 < H^{**}$, then, in addition to mesons, the scalar diquarks are stable particles in this phase (the pseudoscalar diquarks are unstable as before). Note that the initial massless NJL model (1) is parametrized by three independent parameters Λ, G_1 and G_2. So, one may expect that estimates H^* and H^{**} from (44) depend on Λ and G_1. However, as it was pointed out just after (44), the quantity $H^{**} \equiv 1.5G_1$ does not depend really on the cutoff parameter Λ. In contrast, H^* depends both on Λ and G_1. In particular, since for the parameter set accepted in sec. II we have $M \approx 0.355$ GeV, one can present in this case the quantity H^* in the following form $H^* \approx 0.660G_1$.

Having a root $\epsilon^s_0 \equiv (M^o_D)^2$ of the equation (42), one can find in the case $H^* < G_2 < H^{**}$ two zeros (with respect to the variable p_0) of the 1PI Green function $\overline{\Gamma}_{\Delta^{s*}_{AA'}\Delta^s_{AA'}}(p_0)$ as well as four zeros of the equation $\det\overline{\Gamma}^s_{AA'}(p_0) = 0$. They provide us with the following two different mass squared of the excitations in each two scalar $\Delta^s_{AA'}, \Delta^{s*}_{AA'}$-diquark system:

$$(M_\Delta)^2 = (M^o_D - 2\mu)^2, \qquad (M_{\Delta^*})^2 = (M^o_D + 2\mu)^2. \tag{46}$$

In particular, for our choice of the model parameters (see sec. II) we have $M_D^o \approx 1.968M$, where $M \approx 0.355$ GeV. Furthermore, if $G_2 \to H_+^*$ then $M_D^o \to 2M$, if $G_2 \to H_-^{**}$ then $M_D^o \to 0$. Since there are nine scalar $\Delta^s_{AA'}$ diquarks as well as nine scalar $\Delta^{s*}_{AA'}$ antidiquarks in our model, we relate M_Δ in (46) to the mass of the diquark nonet with the baryon number $B = 2/3$ and M_{Δ^*} to the mass of the antidiquark nonet with $B = -2/3$. The difference between diquark and antidiquark masses in (46) is explained by the absence of a charge conjugation symmetry in the presence of a chemical potential μ.

Finally, if $H^{**} < G_2$, then a SU(3)$_{L+R}\times$SU(3)$_c\times$U(1)$_B$ symmetric ground state, i.e. the chirally broken phase of quark matter, is not allowed to exist in the model. The matter is that in this case tachyon singularities of the scalar diquark propagator alone (at $H^{**} < G_2 < H^{***}$), or both of the scalar and pseudoscalar diquark propagators (at $H^{***} < G_2$) appear. As a result, in this case the ground state of the CFL phase is always deeper in comparision with the ground state of the chirally broken phase. So, only the CFL phase may be realized in the model at sufficiently high values of the coupling constant G_2 and arbitrary values of μ. As a consequence, one must expect that at $G_2 \to H_-^{**}$ the critical value μ_c of the chemical potential tends to zero. The fact that at $G_2 \to H_-^{**}$ the diquark mass M_D^o tends to zero may be considered as a precursor, which appears in the chirally broken phase, of the spontaneous breaking of the SU(3)$_c$ symmetry, taking part at $G_2 = H^{**}$.

V. SUMMARY AND DISCUSSION

In the present paper we have continued the investigation, started in our previous paper [9], of the bosonic excitations (mesons and diquarks) of the dense quark matter, composed of u, d, and s quarks, at zero temperature. The consideration is performed in the framework of the massless NJL model (1), omitting the ‘t Hooft six-quark interaction term, for simplicity. In this case, the initial symmetry group of the model, i.e. SU(3)$_L\times$SU(3)$_R\times$SU(3)$_c\times$U(1)$_B\times$U(1)$_A$ does contain the axial U(1)$_A$ subgroup. As a result, we have shown for the model parameter set accepted in sec. II that at sufficiently low values of μ, $\mu < \mu_c \approx 330$ MeV, the chirally broken quark matter phase with SU(3)$_{L+R}\times$SU(3)$_c\times$U(1)$_B$-ground state symmetry is realized and nine massless pseudoscalar mesons (which are the NG bosons), $\pi^\pm$, π^0, K^0, $\bar{K}^0$, $K^\pm$, η_8 and η', appear. (In massless QCD, where U(1)$_A$ is broken on the quantum level, or in NJL models with ‘t Hooft interaction the η'-meson is not a NG boson.)

At $\mu > \mu_c$ the original symmetry is spontaneously broken down to SU(3)$_{L+R+c}$, and the CFL phase does occur. In this case, in accordance with the Goldstone theorem, eighteen NG bosons must appear in the mass spectrum of the model (1). (In contrast, due to the absence of the unphysical U(1)$_A$ symmetry, only seventeen NG bosons must appear in massless QCD.) Considering 1PI Green functions, we have found nine NG bosons in the sector of scalar diquark excitations. Eight of them have to be considered as non-physical, since in real QCD they supply masses to gluons by the Anderson – Higgs mechanism. The remaining scalar NG boson corresponds to a spontaneous breaking of the baryon U(1)$_B$ symmetry. The other nine NG bosons are no more pseudoscalar mesons, but now the massless excitations in the pseudoscalar diquark sector of the model. All that, i.e. the NG boson structure of the model (1), is the main result of the paper [9] which is also confirmed in the present consideration.

Besides NG diquarks, we have proved in [9] the existence of massive diquark excitations in the CFL phase. However, a detailed numerical analysis of the diquark masses vs chemical potential μ was not done there. In the present paper we have argued that all massive diquark excitations of the CFL phase are resonances, since the corresponding singularities of their Green functions in momentum space lie on the second energy Riemann sheet. Moreover, they form scalar- and pseudoscalar SU(3)$_{L+R+c}$ octets and singlets. The mass and width of scalar- and pseudoscalar diquark resonances from octets vs μ have been obtained numerically (see Fig. 3). They are avaluated around 230 MeV and 50 MeV, correspondingly, i.e. this quantities are at least five times smaller, than the mass and width of the scalar diquark resonance in the color superconducting quark matter composed of u and d quarks [11, 12, 13]. (Due to numerical difficulties, we were not able to evaluate the parameters of the above mentioned massive scalar- and pseudoscalar singlet diquark resonances of the CFL phase, however, we guess that their masses are of the same order in magnitude as the masses of the diquark octets.)

To get a more complete view about the diquark properties in the framework of the NJL model (1), we have considered their masses in the chirally broken phase of quark matter, too. It follows from our analysis that i) at sufficiently strong interaction in the diquark channel, i.e. at $G_2 > H^{**} = 1.5G_1$, the existence of this phase is prohibited in the framework of the NJL model (1), ii) depending on the coupling constant G_2, scalar and pseudoscalar diquarks have different properties in this phase. Indeed, at $G_2 < H^*$, where H^* is given in (44), both types of diquarks are resonances. However, at $H^* < G_2 < H^{**}$ the pseudoscalar diquarks remain to be resonances, whereas scalar diquarks are yet stable particles. As this takes place, there is a splitting between the scalar diquark and scalar antidiquark masses (see (46)), which is explained by the violation of the charge conjugation symmetry in the presence of a chemical potential. (Of course, in the chirally broken phase all observable particles are colorless, so one should expect that colored diquarks are confined within baryons (see e.g. [18]). Thus, one may look at our investigation of the diquark masses in the chirally broken phase as an indication of the existence of rather strong quark-quark correlations inside baryons, which might help in a better understanding of baryon dynamics.)

Finally, we have considered in the model (1) the masses of mesons which are stable particles in both phases. In the chirally broken phase, i.e. at $\mu < \mu_c$, all nine pseudoscalar mesons are NG bosons, whereas the nine scalar mesons have equal mass $\equiv 2M \sim 710$ MeV (for the parameter set of the model accepted in sec. II). In the CFL phase, i.e. at $\mu > \mu_c$, these nonet representations of mesons, reducible with respect to the $\mathrm{SU}(3)_{L+R+c}$ group, are decomposed into the octet and singlet representations, each with its own mass. The reason for this octet-singlet mass splitting of mesons is just the color-flavor locked symmetry breaking taking place at $\mu > \mu_c$. In the CFL phase the masses of both types of mesons vary in the interval 300÷900 MeV, when μ varies from 330 MeV to 500 MeV (see Figs 1,2). However, the mass splitting among the scalar and pseudoscalar mesons occurs in different ways. Indeed, as it is easily seen from Fig. 1, the mass of the scalar octet mesons is larger than the mass of the scalar singlet meson, whereas for pseudoscalar mesons the opposite situation takes place (see Fig. 2).

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APPENDIX A: SOME FORMULAE

The present Appendix contains some useful formulae employed in the text.

i) Determinant:

$$\det\begin{pmatrix} A\ , & B \\ C\ , & D \end{pmatrix} = \det[-CB + CAC^{-1}D] = \det[DA - DBD^{-1}C]. \tag{A1}$$

ii) Inverse matrix:

$$\begin{pmatrix} A\ , & B \\ C\ , & D \end{pmatrix}^{-1} = \begin{pmatrix} C^{-1}DL\ , & -N \\ -L\ , & B^{-1}AN \end{pmatrix} = \begin{pmatrix} \bar{L}\ , & -A^{-1}B\bar{N} \\ -D^{-1}C\bar{L}\ , & \bar{N} \end{pmatrix}, \tag{A2}$$

where

$$L = [AC^{-1}D - B]^{-1}\ \ ,\ \ N = [DB^{-1}A - C]^{-1}\ \ ,\ \ \bar{L} = [A - BD^{-1}C]^{-1}\ \ ,\ \ \bar{N} = [D - CA^{-1}B]^{-1}. \tag{A3}$$

iii) Variational derivatives: Let A, B are some operators in the coordinate space with matrix elements $A(x,y) \equiv A(x-y)$ and $B(x,y) \equiv B(x-y)$, respectively. Moreover, let $\sigma(x)$ and $\phi(x)$ are some fields. Then,

$$\mathrm{Tr}\{A\sigma B\phi\} \equiv \int dxdydzduA(x,z)\sigma(z)\delta(z-y)B(y,u)\phi(u)\delta(u-x) = \int dxdyA(x,y)\sigma(y)B(y,x)\phi(x). \tag{A4}$$

It follows from (A4) that

$$\Gamma(x-y) \equiv \frac{\delta^2\mathrm{Tr}\{A\sigma B\phi\}}{\delta\sigma(y)\delta\phi(x)} = A(x,y)B(y,x) = A(x-y)B(y-x). \tag{A5}$$

iv) Fourier transformations: For arbitrary function $F(z)$ it is possible to define the Fourier transformation $\overline{F}(p)$ by the relation

$$\overline{F}(p) = \int d^4zF(z)e^{ipz}, \quad \text{i. e.} \quad F(z) = \int \frac{d^4p}{(2\pi)^4}\overline{F}(p)e^{-ipz}. \tag{A6}$$

Taking these relations into account, one obtains from (A5) that

$$\overline{\Gamma}(p) = \int \frac{d^4q}{(2\pi)^4}\overline{A}(q+p)\overline{B}(q), \tag{A7}$$

where $\overline{A}(q), \overline{B}(q)$ are Fourier transformations of the functions $A(x)$ and $B(x)$, correspondingly.

APPENDIX B: QUARK PROPAGATOR MATRIX

In the Nambu–Gorkov representation the inverse quark propagator matrix S_0^{-1} is given in (7). Using the techniques, elaborated in [10, 11, 12, 13, 14], it is possible to obtain the following expressions for the matrix elements of the quark propagator matrix $S_0 \equiv \begin{pmatrix} S_{11}, S_{12} \\ S_{21}, S_{22} \end{pmatrix}$:

$$S_{11}(x-y) = \int \frac{d^4q}{(2\pi)^4}\, e^{-iq(x-y)} \left\{ \frac{q_0 - E^+}{q_0^2 - (E_{B\Delta}^+)^2}\gamma^0\bar\Lambda_+ + \frac{q_0 + E^-}{q_0^2 - (E_{B\Delta}^-)^2}\gamma^0\bar\Lambda_- \right\}, \tag{B1}$$

$$S_{12}(x-y) = -i\Delta B \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{1}{q_0^2 - (E_{B\Delta}^+)^2}\gamma^5\bar\Lambda_- + \frac{1}{q_0^2 - (E_{B\Delta}^-)^2}\gamma^5\bar\Lambda_+ \right\}, \tag{B2}$$

$$S_{21}(x-y) = -i\Delta^* B \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{1}{q_0^2 - (E_{B\Delta}^+)^2}\gamma^5\bar\Lambda_+ + \frac{1}{q_0^2 - (E_{B\Delta}^-)^2}\gamma^5\bar\Lambda_- \right\}, \tag{B3}$$

$$S_{22}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{q_0 + E^+}{q_0^2 - (E_{B\Delta}^+)^2}\gamma^0\bar\Lambda_- + \frac{q_0 - E^-}{q_0^2 - (E_{B\Delta}^-)^2}\gamma^0\bar\Lambda_+ \right\}, \tag{B4}$$

where $M = \sqrt{\frac{2}{3}}\sigma$, $\bar\Lambda_\pm = \frac{1}{2}(1 \pm \frac{\gamma^0(\vec\gamma\vec q - M)}{E})$. Moreover, $(E_{B\Delta}^\pm)^2 = (E^\pm)^2 + |\Delta|^2 B^2$, $E^\pm = E \pm \mu$, $E = \sqrt{\vec q^2 + M^2}$ and $B = \sum_{A=2,5,7} \tau_A \lambda_A$. (In these and other similar expressions, q_0 is a shorthand notation for $q_0 + i\varepsilon \cdot \mathrm{sgn}(q_0)$, where the limit $\varepsilon \to 0_+$ must be taken at the end of all calculations. This prescription correctly implements the role of μ as the chemical potential and preserves the causality of the theory.) It is clear from (B1)-(B4) that all color- and flavor dependences in the matrix elements S_{11}, S_{12}, S_{21} and S_{22} arise only due to the matrix B. It is a 9×9 matrix in the nine-dimensional space $c \times f$ which is the direct production of color and flavor spaces. Note, in the chirally broken quark matter phase, where $\Delta = 0$, $M \neq 0$, the expressions for the matrix elements (B1)-(B4) have a simpler form. Namely, it is clear that in this phase $S_{12}(x-y) = S_{21}(x-y) = 0$ and

$$S_{11}(x-y) = \mathbf{1_c} \times \mathbf{1_f} \times \int \frac{d^4q}{(2\pi)^4}\, e^{-iq(x-y)} \left\{ \frac{\gamma^0\bar\Lambda_+}{q_0 + E^+} + \frac{\gamma^0\bar\Lambda_-}{q_0 - E^-} \right\}, \tag{B5}$$

$$S_{22}(x-y) = \mathbf{1_c} \times \mathbf{1_f} \times \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^0\bar\Lambda_-}{q_0 - E^+} + \frac{\gamma^0\bar\Lambda_+}{q_0 + E^-} \right\}, \tag{B6}$$

i.e. the matrix elements (B5)-(B6) are proportional to the unit matrices both in the color and flavor spaces.

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