

Mesons and diquarks in the CFL phase of dense quark matter

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Abstract

The spectrum of meson and diquark excitations of the color–flavor locked (CFL) phase of dense quark matter is considered in the framework of the Nambu – Jona-Lasinio model. We have found that in this phase all Nambu–Goldstone bosons are realized as scalar and pseudoscalar diquarks. Other diquark excitations are resonances with mass value around 230 MeV. Mesons are stable particles in the CFL phase. Their masses vs chemical potential lie in the interval 300÷500 MeV.

It is well-known that at asymptotically high baryon densities the ground state of massless three-flavor QCD corresponds to the so-called color – flavor locked (CFL) phase [1]. One of the most noticeable differences between color superconductivity phenomena with three and two quark species is that the CFL effect is characterized by a hierarchy of energy scales. Indeed, at the lowest scale of this phase lie NG bosons, which dominate in all physical processes with energies smaller than the superconducting gap Δ . Evident contributors at higher energy scales are quark quasiparticles, which in the CFL phase have an energy greater than Δ . However, up to now we know much less about other excitations, whose energy and mass are of the order of Δ in magnitude. Among these particles are ordinary scalar and pseudoscalar mesons, massive diquarks etc, i.e. particles which might play an essential role in dynamical processes of the CFL phase. Here we are going to discuss just this type of excitations of the CFL ground state, i.e. mesons and massive diquarks, in the framework of the massless three-flavor NJL model with Lagrangian

$$L = \bar{q}[\gamma^\nu i\partial_\nu + \mu\gamma^0]q + G_1 \sum_{a=0}^8 [(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma^5\tau_a q)^2] + G_2 \sum_{A=2,5,7} \sum_{A'=2,5,7} \{[\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q][\bar{q}i\gamma^5\tau_A\lambda_{A'}q^C] + [\bar{q}^C\tau_A\lambda_{A'}q][\bar{q}\tau_A\lambda_{A'}q^C]\}. \quad (1)$$

In (1), $\mu \geq 0$ is the quark number chemical potential which is the same for all quark flavors, $q^C = C\bar{q}^t$, $\bar{q}^C = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). The quark field q is a flavor and color triplet as well as a four-component Dirac spinor. Furthermore, we use the notations τ_a, λ_a for Gell-Mann matrices in the flavor and color space, respectively ($a = 1, \dots, 8$); $\tau_0 = \sqrt{\frac{2}{3}} \cdot \mathbf{1}_f$ is proportional to the unit matrix in the flavor space. Clearly, the Lagrangian (1) as a whole is invariant under transformations from the color group $SU(3)_c$. In addition, it is symmetric under the chiral group $SU(3)_L \times SU(3)_R$ as well as under the

baryon-number conservation group $U(1)_B$ and the axial group $U(1)_A$.¹ In all numerical calculations below, we used the following values of the model parameters: $\Lambda = 602.3$ MeV, $G_1\Lambda^2 = 2.319$ and $G_2 = 3G_1/4$, where Λ is an ultraviolet cutoff parameter in the three-dimensional momentum space.

Introducing collective scalar $\sigma_a(x)$, $\Delta_{AA'}^s(x)$ and pseudoscalar $\pi_a(x)$, $\Delta_{AA'}^p(x)$ fields,

$$\begin{aligned}\sigma_a(x) &= -2G_1(\bar{q}\tau_a q), & \Delta_{AA'}^s(x) &= -2G_2(\bar{q}^C i\gamma^5 \tau_A \lambda_{A'} q), \\ \pi_a(x) &= -2G_1(\bar{q}i\gamma^5 \tau_a q), & \Delta_{AA'}^p(x) &= -2G_2(\bar{q}^C \tau_A \lambda_{A'} q),\end{aligned}\quad (2)$$

($a = 0, 1, \dots, 8$; $A, A' = 2, 5, 7$) and then integrating out quark fields from the theory, it is possible to obtain the following generating functional of the two-point one-particle irreducible (1PI) Green functions of the mesons and diquarks in the CFL phase

$$\begin{aligned}\mathcal{S}_{\text{eff}}(\sigma_a, \pi_a, \Delta_{AA'}^{s,p}, \Delta_{AA'}^{s,p*}) &= - \int d^4x \left[\frac{\sigma_a^2 + \pi_a^2}{4G_1} + \frac{\Delta_{AA'}^s \Delta_{AA'}^{s*} + \Delta_{AA'}^p \Delta_{AA'}^{p*}}{4G_2} \right] + \\ &\frac{i}{4} \text{Tr} \left\{ S_0 \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^t \end{pmatrix} S_0 \begin{pmatrix} \Sigma & K \\ K^* & \Sigma^t \end{pmatrix} \right\},\end{aligned}\quad (3)$$

where

$$\begin{aligned}\Sigma &= \tau_a \sigma_a + i\gamma^5 \pi_a \tau_a, & K &= (\Delta_{AA'}^p + i\Delta_{AA'}^s \gamma^5) \tau_A \lambda_{A'}, \\ \Sigma^t &= \tau_a^t \sigma_a + i\gamma^5 \pi_a \tau_a^t, & K^* &= (\Delta_{AA'}^{p*} + i\Delta_{AA'}^{s*} \gamma^5) \tau_A \lambda_{A'},\end{aligned}\quad (4)$$

S_0 is the Nambu – Gorkov representation for the quark propagator in this phase,

$$S_0^{-1} = \begin{pmatrix} i\gamma^\nu \partial_\nu + \mu\gamma^0, & -i\Delta\gamma^5(\tau_2\lambda_2 + \tau_5\lambda_5 + \tau_7\lambda_7) \\ -i\Delta\gamma^5(\tau_2\lambda_2 + \tau_5\lambda_5 + \tau_7\lambda_7), & i\gamma^\nu \partial_\nu - \mu\gamma^0 \end{pmatrix},\quad (5)$$

and Δ is the gap parameter. Using the expression (3), one can find the 1PI Green functions for mesons and diquarks in the CFL phase, namely

$$\Gamma_{XY}(x-y) = -\frac{\delta^2 \mathcal{S}_{\text{eff}}^{(2)}}{\delta Y(y) \delta X(x)},\quad (6)$$

where $X(x), Y(x) = \sigma_a(x), \pi_b(x), \Delta_{AA'}^{s,p}(x), \Delta_{BB'}^{s,p*}(x)$. In total, the Green functions (6) form a 54×54 matrix which is, fortunately, a reducible one. It is well known that in the rest frame of the momentum space representation, i.e. at $p = (p_0, 0, 0, 0)$, the meson and diquark masses are the zeros of the determinant of this matrix. So, after tedious both analytical and numerical calculations (the details are presented in [2]) we have obtained the following results on the mass spectrum of the bosonic excitations of the CFL phase.

In the Figs 1,2 the mass behavior for the scalar and pseudoscalar mesons is presented in the CFL phase. It is easily seen that i) there is a mass splitting between octet and singlet of mesons, ii) the singlet mass is smaller than octet one for scalar mesons, but for pseudoscalar mesons the situation is inverse, iii) in the CFL phase the meson masses are greater than 300 MeV.

¹In a more realistic case, the additional 't Hooft six-quark interaction term should be taken into account in order to break the axial $U(1)_A$ symmetry.

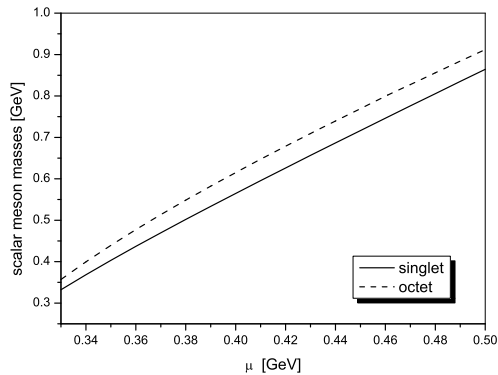


Figure 1: The behavior of the scalar meson masses vs μ in the CFL phase.

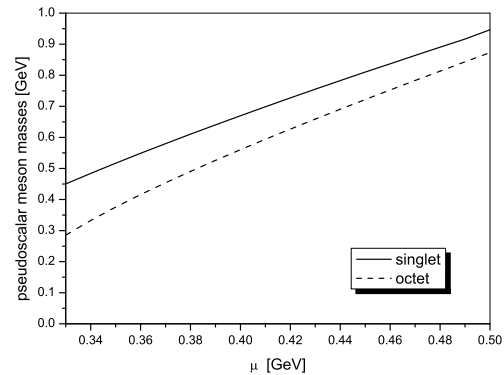


Figure 2: The behavior of the pseudoscalar meson masses in the CFL phase.

In the diquark sector we have found 18 Nambu-Goldstone bosons. Moreover, there are a scalar octet and singlet as well as pseudoscalar octet and singlet of nontrivial diquark excitations of the CFL phase. The diquarks from scalar and pseudoscalar octets are resonances with mass around 230 MeV. The properties of diquarks in the CFL phase were also considered earlier in the papers [2, 3].

For comparison, let us mention the existence of nontrivial excitations of the color superconducting (2SC) phase of quark matter with two quark flavors. In this phase the masses of σ and π mesons lie in the same interval $300 \div 500$ MeV as the meson masses in the CFL phase. But the scalar diquark is a very heavy resonance with mass ~ 1100 MeV [4]. If the electric charge neutrality constraint is imposed, then in the 2SC phase diquark is a stable particle with mass ~ 200 MeV [5].

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